

APPLIED MATHEMATICS

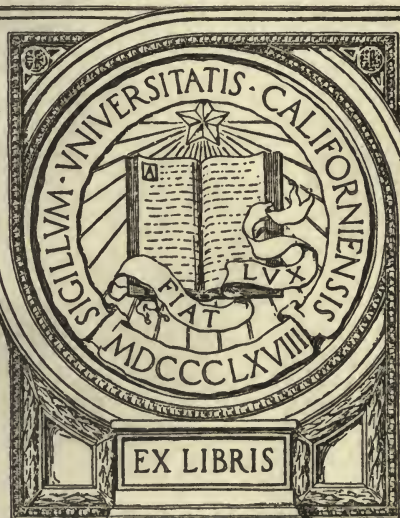
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APPLIED MATHEMATICS

FOR

JUNIOR HIGH SCHOOLS

AND

HIGH SCHOOLS

BY

EUGENE HENRY BARKER

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PREFACE

THIS book has been written to meet the demand for a practical course in applied mathematics which shall coördinate the schoolroom lesson and the actual problem of the industrial and commercial world. It presents the body of mathematical information which is likely to be of daily service, no matter what one's occupation may be.

Applied Mathematics is the outgrowth of twelve years of experimentation in a high school of over two thousand students, and the exercises in the book have stood the test of classroom trial. The examples and problems have been selected from many fields—industrial, commercial, mechanical, agricultural—and, as far as possible, are such as occur in the household, on the farm, in the factory, and at the office. The book gives in teachable form:

(1) An adequate treatment of the fundamental operations of arithmetic, with proper attention to modern methods of performing these operations.

(2) A consideration of ordinary business transactions.

(3) A sufficient acquaintance with the symbols of algebra and of algebraic operations to enable the student to interpret and apply simple formulas.

(4) Enough geometry to enable him to compute the areas and volumes of the common geometric figures.

(5) A study of graphic charts and their use.

(6) Training in the use of mathematical tables.

(7) Practice in the power to judge a computed result with reference to its reasonableness.

Having drawn on various sources for materials, the author takes this opportunity to express his indebtedness to numerous interested teachers and technical workers whose criticisms and advice have been of invaluable service in the preparation of the book. Especially does he desire to acknowledge obligation to his friend Dr. Nathaniel L. Gardner, of the University of California, for kindly assistance and sympathetic encouragement.

E. H. B.

SAN FRANCISCO, CALIFORNIA,
March, 1920.

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APPLIED MATHEMATICS

CHAPTER I

ADDITION AND SUBTRACTION

Addition.—The greater part of all practical calculation is addition. It is of the utmost importance that you add with ease, precision, and speed, but skill in this comes only by much practice. This book aims to give you both practice and principle—problems to solve and methods of solving them.

Addition is the process of finding a number which is equal to two or more numbers taken together. The result obtained by adding is called the *sum*.

First Method

Rule.—Group or combine two or more numbers (mentally) and name results only.

EXAMPLE				In the example given, instead of saying “seven and two are nine, and seven are sixteen,” and so on, omit as far as possible all unnecessary words and repetitions.
6	4	7		Say simply, or rather <i>think</i> (first column), “nine, sixteen, twenty-four, thirty-two, thirty-six; (second column), seven, sixteen, twenty-five, thirty-one, thirty-five, thirty-eight; (third column), nine, fourteen, twenty-one, twenty-nine, thirty-five, <i>forty-three</i> .”
5	9	2		
7	9	7		
8	6	8		
6	4	8		
8	3	4		
4	3	8	6	

Be constantly on the alert for simple combinations of numbers and thus lessen the labor of addition. For instance, if you have a 6 and a 5 in a column, you know that they make 11, and if your sum up to that point is 27, it is much easier to think 38 than it is to take the two intermediate steps, each of which is more difficult than the complete step.

Always add the number to be carried to the next column on *commencing*, and if the columns are long, write it down, as it may save the trouble of going over work already performed. Take pride and pains in making neat, plain, and legible figures.

*Check.*¹ — Add in the opposite direction, upward or downward.

This is preferable to doing the work over a second time in the same way, as a mistake once made is likely to be repeated.

Exercise

Add:

1.	2.	3.	4.
4 5 3 6	5 6 4 7	6 7 4 3	6 5 7 8
5 6 5 4	6 4 8 6	6 5 9 7	7 4 9 7
5 2 3 4	5 4 8 7	8 5 4 6	7 8 7 8
6 8 7 8	6 5 4 3	7 8 8 9	7 7 8 8
3 4 2 5	6 4 6 7	5 6 4 3	5 4 8 3
5 4 6 3	6 4 4 4	6 5 9 4	8 1 5 5
5 4 7 3	7 5 8 4	8 6 8 0	8 5 4 2
<u>7 5 1 7</u>	<u>7 1 1 8</u>	<u>7 8 7 6</u>	<u>7 5 4 0</u>
5.	6.	7.	8.
5 6 6 8	7 4 6 3	6 5 9 5	6 4 3 2
5 3 2 6	6 4 9 6	6 4 9 8	7 5 9 1
6 4 8 0	8 5 9 1	7 4 4 5	7 5 3 0
7 5 4 2	7 6 6 4	8 5 4 3	6 7 4 9
6 4 4 9	7 6 9 7	7 5 9 4	6 4 3 4
6 5 5 8	7 5 4 9	6 5 9 6	4 5 7 2
7 7 9 4	5 4 8 3	5 4 8 6	7 4 8 6
<u>6 4 5 3</u>	<u>5 4 8 6</u>	<u>6 4 2 1</u>	<u>7 4 4 2</u>

¹ Always check your work, no matter how expert you are in computation.

9.				10.				11.				12.			
4	6	5	6	7	6	4	5	2	7	8	5	5	9	4	6
6	5	1	9	8	6	5	6	3	4	9	1	6	4	8	2
6	4	7	5	8	6	6	9	3	4	4	4	6	6	7	7
7	6	4	5	7	8	8	7	4	3	2	2	6	4	4	1
6	4	3	8	8	6	8	6	5	4	9	6	7	4	3	2
7	7	5	5	7	8	8	6	5	6	6	3	6	7	7	3
6	7	7	0	6	0	1	8	6	0	6	2	6	4	5	7
6	8	5	4	5	7	7	8	5	6	2	4	5	3	8	4

13. From the following figures determine the value of Alaska's output during the last fifteen years :

Gold	\$ 260,488,175
Silver	3,011,153
Copper	46,281,148
Other minerals	2,751,638
Fisheries and furs	299,581,890
Other merchandise	<u>1,705,820</u>

14. From the following data find the total attendance and total receipts for the World's Championship Series for the fourteen years given :

YEAR	CLUBS	GAMES	ATTENDANCE	RECEIPTS
1905	New York-Athletics	5	91,723	\$ 68,436.00
1906	White Sox-Cubs	6	99,845	106,550.00
1907	Chicago-Detroit	5	78,068	101,728.00
1908	Chicago-Detroit	5	62,223	94,975.00
1909	Pittsburgh-Detroit	7	145,295	188,302.00
1910	Athletics-Chicago	5	125,222	173,980.00
1911	Athletics-Giants	6	179,851	342,364.00
1912	Red Sox-Giants	8	252,037	490,833.00
1913	Athletics-Giants	5	150,992	325,980.00
1914	Boston-Philadelphia	4	111,009	226,739.00
1915	Boston-Philadelphia	5	143,351	320,361.50
1916	Boston-Brooklyn	5	162,859	385,590.50
1917	Chicago-New York	6	186,654	425,878.00
1918	Boston-Chicago	6	128,483	179,619.00

15. Find the totals in the following table showing the local cut of timber from the National forests in a recent year:

STATE	CUT UNDER SALE	CUT UNDER FREE USE	
		Written Permit	Without Written Permit
	<i>Board Feet</i>	<i>Board Feet</i>	<i>Board Feet</i>
Oregon	88,157,000	7,663,000	2,297,000
Idaho	76,679,000	20,969,000	74,000
California	57,352,000	7,711,000	192,000
Montana	47,249,000	8,861,000	1,774,000
Arizona	44,672,000	1,224,000	4,889,000
Washington	48,187,000	1,918,000	50,000
Alaska	35,019,000	11,723,000	3,000,000
Colorado	39,611,000	9,627,000	33,000
Utah	25,844,000	9,727,000	126,000
New Mexico	17,761,000	4,479,000	9,270,000
Wyoming	20,513,000	7,382,000	
South Dakota	20,896,000	5,224,000	
Arkansas	12,238,000	138,000	
Minnesota	5,323,000	329,000	10,000

ORAL DRILL

In each of the following examples find by inspection the sum of the two amounts given:

- | | | |
|----------------------|----------------------|----------------------|
| 1. \$ 2.50, \$.50 | 11. \$ 1.25, \$ 8.75 | 21. \$ 5.50, \$ 4.50 |
| 2. \$ 1.25, \$.75 | 12. \$ 2.10, \$ 4.65 | 22. \$ 6.50, \$ 4.35 |
| 3. \$ 2.15, \$.35 | 13. \$ 8.40, \$ 1.60 | 23. \$ 8.40, \$ 8.75 |
| 4. \$ 3.75, \$ 1.10 | 14. \$ 7.65, \$ 2.35 | 24. \$ 7.15, \$ 9.90 |
| 5. \$ 4.00, \$ 2.35 | 15. \$ 3.15, \$ 3.75 | 25. \$ 4.45, \$ 5.40 |
| 6. \$ 3.70, \$ 2.45 | 16. \$ 4.15, \$ 6.10 | 26. \$ 9.10, \$ 6.85 |
| 7. \$ 7.20, \$ 3.75 | 17. \$ 4.65, \$ 9.15 | 27. \$ 8.40, \$ 4.65 |
| 8. \$ 8.15, \$ 4.15 | 18. \$ 3.65, \$ 4.30 | 28. \$ 4.10, \$ 7.40 |
| 9. \$ 2.65, \$ 5.65 | 19. \$ 4.10, \$ 6.20 | 29. \$ 5.30, \$ 6.75 |
| 10. \$ 3.35, \$ 6.35 | 20. \$ 5.20, \$ 9.70 | 30. \$ 2.75, \$ 4.75 |

Exercise

1. The sales of a merchant for six days amounted to the following: \$73.55, \$64.80, \$94.40, \$89.35, \$95.75, \$111.10. What was the total amount for the week?



STANDING TIMBER IN WASHINGTON

2. From the data on the preceding page, find the annual total of timber cut under sale in the states of Oregon, Idaho, California, and Washington.

3. The sales of the Springfield Department Store for the four weeks of December amounted to \$32,365.45, \$38,425.55, \$41,285.25, \$54,321.10. What was the total amount?

4. New York is 1405 miles east of Omaha, and San Francisco is 1864 miles west of Omaha. How far is it from New York to San Francisco by way of Omaha?

5. The sales of the Western Gun Co. for the twelve months in a year were as follows: \$3771.60, \$4236.40, \$3815.75, \$4625.50, \$3275.85, \$3018.95, \$3885.40, \$4129.25, \$3828.35, \$4121.70, \$4333.55, \$4963.00. What was the total amount?

6. What is the area of New England in square miles if the area of Maine in square miles is 33,040; of New Hampshire, 9305; of Vermont, 9565; of Massachusetts, 8315; of Rhode Island, 1250; of Connecticut, 4990?

7. The area in square miles of the main body of the United States is 3,088,519; the outlying possessions are Alaska, 590,884 square miles; Hawaii, 6449; Porto Rico, 3435; Philippines, 115,026; other outlying possessions, 761 square miles. Find the total area of the possessions.

8. Approximately the areas of the Great Lakes in square miles are as follows: Superior, 32,000; Michigan, 22,400; Huron, 23,000; Erie, 10,000; Ontario, 6700. What is the total area?

9. If in Thomaston, 3980 clocks were made during the first week in September, 3986 the second week, 4016 the third week, and 4221 the fourth week, how many clocks were made there during those four weeks?

10. The annual production of corn in the United States for a number of years averaged \$854,000,000; of hay, \$467,000,000; the production of cotton, \$406,000,000; of wheat, \$395,000,000; and of oats, \$254,000,000. Find the total average annual value of these five productions.

Second Method

Rule. — Set down the sums of the numbers in each column, each in its place, and add these sums.

EXAMPLE

4	6	2	7
8	4	3	6
8	2	7	6
4	9	3	7
5	4	6	4
7	6	4	9
		3	9
		2	5
	3	1	
3	6		
3	9	3	8 9

The addition can be begun either at the right or at the left and the partial sums thus obtained added. The result is of course the same in either case.

EXAMPLE

4	6	2	7
8	4	3	6
8	2	7	6
4	9	3	7
5	4	6	4
7	6	4	9
3	6		
	3	1	
		2	5
			3 9
3	9	3	8 9

Exercise

Add:

1.	2.	3.	4.
4 7 9 4	5 6 4 7	6 7 5 8	8 6 9 4
6 4 5 8	7 5 9 4	7 5 4 2	7 5 2 5
7 6 8 6	6 4 3 6	7 5 6 6	8 5 5 5
6 7 8 5	5 6 4 3	8 6 9 0	7 5 4 5
5 3 3 8	6 5 8 3	7 5 4 7	7 5 9 8
7 8 8 5	6 4 9 8	7 6 6 4	8 8 6 5
7 8 5 0	7 0 8 8	8 7 8 6	7 6 7 0
8 6 0 8	5 5 4 2	9 0 5 7	8 0 4 9
5.	6.	7.	8.
7 6 7 8	5 4 8 7	4 9 0 4	6 0 8 8
6 5 5 8	8 6 7 4	6 5 9 8	5 8 6 6
5 4 9 0	6 4 7 6	6 3 2 8	8 0 6 3
6 7 4 8	8 6 7 4	4 9 7 5	9 7 6 8
7 5 6 8	6 7 5 7	5 8 8 7	5 4 4 5
8 8 7 9	7 6 9 4	6 5 8 7	7 6 6 9
8 5 4 8	7 4 2 0	8 0 1 5	6 1 8 5
7 5 1 7	6 4 8 8	7 5 4 3	7 4 4 8

9. Find from the figures given below the world's total coffee production for a year :

COUNTRY PRODUCING	QUANTITY Pounds
Brazil	1,490,715,000
Central America	231,315,000
Venezuela	121,350,000
Colombia	136,500,000
Dutch East Indies	63,799,000
Haiti	57,594,000
Mexico	53,759,000
United States	48,179,000
British India	39,973,000
Jamaica	10,034,000
Other countries	333,272,000



OIL WELLS NEAR BAKERSFIELD, CALIFORNIA

10. Find from the figures given on page 9 the world's total production of petroleum for an average year :

COUNTRY	PRODUCTION Bbl., 42 gal.
United States	281,104,104
Russia	68,548,062
Mexico	32,910,508
Rumania	12,029,913
Dutch East Indies	12,386,808
India	8,202,674
Galicia	4,158,899
Japan and Formosa	3,118,464
Peru	2,487,251
Germany	995,764
Egypt	221,768
Trinidad	750,000
Canada	215,464
Italy	39,548
Other countries	10,000

11. Find from the following figures the total United States cotton crop :

STATE	BALES
North Carolina	791,376
South Carolina	1,241,551
Georgia	2,113,459
Alabama	1,099,843
Mississippi	1,008,654
Louisiana	369,146
Texas	3,299,795
Arkansas	845,014
Tennessee	349,555
Oklahoma	671,684
Other states	173,370

12. From the following figures, find the world's total railway mileage :

COUNTRY	MILEAGE
Europe	215,140
America	354,248
Asia	68,198
Africa	27,531
Australia, etc.	22,006

13. Find from the following figures the total number of libraries, and the total number of volumes in the libraries of the North Atlantic States :

STATES	LIBRARIES	VOLUMES
North Atlantic Division :		
Maine	190	1,315,211
New Hampshire	212	1,344,522
Vermont	139	782,961
Massachusetts	626	10,596,707
Rhode Island	87	1,269,135
Connecticut	249	3,279,705
New York	1,037	13,308,082
New Jersey	213	2,481,082
Pennsylvania	446	6,112,381

Exercise

Try the following examples by each of the methods given. Time yourself to see at what method you are most expert and accurate. Seek to perfect yourself in the use of that method.

To gain speed and accuracy in adding by groups requires constant practice. This develops a power which is of greatest service in most of the arithmetic the average person is called on to do.

Add :

1.	2.	3.	4.	5.	6.
6847	5735	3492	6043	5678	6504
5793	6507	6043	8704	8654	3863
8224	8085	8506	7427	7435	5496
6372	7507	7438	5490	8538	8179
3862	5970	8542	6302	8475	7496
9734	5396	7506	3075	7564	8675
6823	6497	8468	6482	6474	9601
<u>7845</u>	<u>9181</u>	<u>7194</u>	<u>7619</u>	<u>7509</u>	<u>9606</u>

7.	8.	9.	10.	11.	12.
7439	7407	6574	7587	9674	9501
7428	8506	5741	7482	8308	7302
6074	8405	6429	8563	8269	7629
9164	7539	7594	8640	8538	8429
8542	9501	7597	7538	7439	7953
5684	8645	6392	7539	7594	9042
8648	7840	7593	7538	8585	7308
<u>7493</u>	<u>8786</u>	<u>7749</u>	<u>6989</u>	<u>8849</u>	<u>8534</u>



A NEW ENGLAND MARBLE QUARRY

13. Seven cars left Rutland, Vt., loaded with marble as follows: The first car, 36,675 pounds; the second, 36,850; the third, 37,200; the fourth, 37,650; the fifth, 37,150; the sixth, 38,090; and the seventh, 27,360. What was the total load?

14. The enrollment in the common schools of certain states was found to be as follows: Illinois, 983,921; Ohio, 827,414; Missouri, 741,745; Iowa, 549,449; Indiana, 538,881; Michigan, 521,463; Wisconsin, 465,490; Minnesota, 429,012; Kansas, 381,595; Nebraska, 279,532; North Dakota, 124,000; South Dakota, 110,094. What was the total enrollment of all the states mentioned?

15. The area of Virginia is 42,450 square miles; of Tennessee, 42,050 square miles; of Maryland, 12,210 square miles; of North Carolina, 10,200 square miles more than Tennessee; and of Louisiana, 6270 square miles more than Virginia. What is the total area of these states?

16. In one day a certain factory at Lowell turned out 17,200 buttons; in another, 18,560; in another, 18,570; in another, 16,500; and in another, 18,250. What was the total output for these five days?

Subtraction

Subtraction is the process of finding the difference between two numbers. The greater number is called the *minuend* and the smaller the *subtrahend*. The *difference* is sometimes called the *remainder*.

EXAMPLE

647	minuend
<u>235</u>	subtrahend
412	difference or remainder

In performing the work it is customary, but not necessary, to write the number to be subtracted under the other number.

Check. — Add the remainder to the subtrahend. The result is the minuend.

ORAL DRILL

In each of the following examples find by inspection the difference between the two amounts given :

- | | | |
|-------------------|---------------------|---------------------|
| 1. \$ 1, \$.75 | 11. \$ 10, \$ 7.05 | 21. \$ 20, \$ 17.60 |
| 2. \$ 2, \$ 1.10 | 12. \$ 10, \$ 6.90 | 22. \$ 20, \$ 17.90 |
| 3. \$ 2, \$.85 | 13. \$ 10, \$ 4.30 | 23. \$ 25, \$ 21.40 |
| 4. \$ 3, \$ 1.90 | 14. \$ 10, \$ 3.40 | 24. \$ 30, \$ 27.15 |
| 5. \$ 3, \$ 2.15 | 15. \$ 10, \$ 7.80 | 25. \$ 30, \$ 28.80 |
| 6. \$ 4, \$ 2.30 | 16. \$ 15, \$ 12.30 | 26. \$ 40, \$ 37.10 |
| 7. \$ 4, \$ 3.15 | 17. \$ 15, \$ 5.80 | 27. \$ 40, \$ 37.60 |
| 8. \$ 4, \$ 3.60 | 18. \$ 15, \$ 11.10 | 28. \$ 50, \$ 42.35 |
| 9. \$ 5, \$ 1.40 | 19. \$ 15, \$ 13.10 | 29. \$ 50, \$ 47.20 |
| 10. \$ 5, \$ 3.60 | 20. \$ 20, \$ 17.10 | 30. \$ 50, \$ 41.15 |

Exercise

Perform the following indicated subtractions :

- | | | | |
|----------------------------|----------------------------|----------------------------|----------------------------|
| 1.
7584
<u>5645</u> | 2.
6748
<u>3769</u> | 3.
7556
<u>5642</u> | 4.
8864
<u>6468</u> |
| 5.
9530
<u>6678</u> | 6.
7765
<u>3456</u> | 7.
6728
<u>4635</u> | 8.
5460
<u>3254</u> |
| 9.
6549
<u>4346</u> | 10.
5643
<u>2647</u> | 11.
8790
<u>4769</u> | 12.
7597
<u>7432</u> |
| 13.
6685
<u>4820</u> | 14.
5453
<u>3759</u> | 15.
8674
<u>6427</u> | 16.
5842
<u>3425</u> |

17. Find the amount by which the 1916-1917 total American wheat crop was less than the 1915-1916 crop, using the following figures :

COUNTRY	NUMBER OF BUSHELS	
	1915-16	1916-17
United States	1,011,000,000	640,000,000
Canada	376,304,000	200,000,000
Argentina	168,000,000	76,800,000
Uruguay	11,000,000	4,800,000
Chile	21,200,000	20,000,000
Mexico	8,000,000	8,000,000



AGRICULTURAL BUILDING, WASHINGTON, D. C.

18. Using the following figures, find the distance saved between San Francisco and New York if the Panama route is taken :

From San Francisco to New York via Straits of Magellan	13,083 mi.
From San Francisco to Liverpool via Straits of Magellan	13,630 mi.
From San Francisco to New York via Panama	5,300 mi.
From San Francisco to Liverpool via Panama	7,954 mi.

Find the distance saved between San Francisco and Liverpool under the same conditions. In each case find the number of days saved if a vessel makes on an average 360 miles a day.

CHAPTER II

MULTIPLICATION AND DIVISION

Multiplication is the process of taking one number as many times as there are units in another. The number taken or multiplied is called the *multiplicand*. The number showing how many times the multiplicand is taken is called the *multiplier*. The result obtained by multiplying is called the *product*.

EXAMPLES

7 multiplicand
5 multiplier

35 product

632 multiplicand
57 multiplier

4424
3160

36024 product

Check. — Divide the product by the multiplier. The result is the multiplicand.

Certain special methods of multiplication are given on succeeding pages. As a preliminary, the following oral and written exercises are suggested :

ORAL DRILL

Perform the following indicated multiplications :

1.	2.	3.	4.	5.	6.
6	8	12	12	9	9
3	4	5	6	7	8

7.	8.	9.	10.	11.	12.
12	13	14	15	16	16
<u>8</u>	<u>2</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>5</u>
13.	14.	15.	16.	17.	18.
18	22	32	44	18	19
<u>2</u>	<u>4</u>	<u>3</u>	<u>3</u>	<u>10</u>	<u>10</u>
19.	20.	21.	22.	23.	24.
10	11	20	18	20	18
<u>13</u>	<u>20</u>	<u>10</u>	<u>6</u>	<u>6</u>	<u>7</u>
25.	26.	27.	28.	29.	30.
60	70	10	31	17	18
<u>70</u>	<u>7</u>	<u>10</u>	<u>11</u>	<u>8</u>	<u>6</u>

Exercise

Perform the following indicated multiplications :

1.	2.	3.	4.	5.
6742	7568	8594	7469	6478
<u>24</u>	<u>32</u>	<u>56</u>	<u>64</u>	<u>243</u>
6.	7.	8.	9.	10.
8534	7597	4378	6464	7474
<u>453</u>	<u>564</u>	<u>434</u>	<u>464</u>	<u>646</u>
11.	12.	13.	14.	15.
8686	6854	7895	7856	4739
<u>468</u>	<u>658</u>	<u>206</u>	<u>725</u>	<u>978</u>
16.	17.	18.	19.	20.
6783	6496	8745	7490	4836
<u>842</u>	<u>4536</u>	<u>7586</u>	<u>4326</u>	<u>4262</u>

21. At \$275 a month, what will a man's yearly salary amount to?

22. A school librarian purchased 24 dictionaries at \$9.75 each. What did they all cost?

23. If a cubic foot of ice weighs 58 pounds, what will 1492 cubic feet of ice weigh?



AN ATLANTIC COASTWISE SAILING VESSEL

24. Two vessels are 8888 miles apart and are sailing toward each other, one at the rate of 144 miles, the other at the rate of 166 miles a day. How far apart will they be at the end of three weeks?

25. Two vessels are 8888 miles apart and are sailing in opposite directions, one at the rate of 144 miles, the other at the rate of 166 miles per day. How far apart will they be at the end of a week?

26. What will 34 automobiles cost at \$1375 apiece?

27. If 76 acres of land are bought at \$ 740 an acre, what is the total cost ?

28. A dealer sells 182 sets of harness at \$ 26 a set. What does he receive ?

29. If in walking a mile a man averages 2112 steps, how many steps will he take in walking 27 miles ?

30. 1972 pickets are required to fence one side of a square field. How many pickets will be required to fence 19 fields of the same size and shape ?

31. In a freight train are 32 cars, each containing 82 barrels of flour, each barrel weighing 196 pounds. How many pounds of flour in the trainload ?

32. An oil well driller charged 35 cents per foot for drilling and 75 cents per foot for the tubing. If the well was drilled 1220 feet and tubed 720 feet, what was the cost ?

Short Methods

There are many combinations of numbers which can be multiplied by certain short methods and the student should always be on the lookout for number combinations which will lessen labor in computation.

Multiplication by 25. Rule. — To multiply by 25, add two ciphers and divide by 4.

EXAMPLE. Multiply 28 by 25.

$$28 \times 25 = 28 \times 100 \div 4 = 700.$$

ORAL DRILL

By this method find the results of the following indicated multiplications :

1.	2.	3.	4.	5.
8	12	16	20	24
<u>25</u>	<u>25</u>	<u>25</u>	<u>25</u>	<u>25</u>

6.	7.	8.	9.	10.
28	32	36	40	44
<u>25</u>	<u>25</u>	<u>25</u>	<u>25</u>	<u>25</u>
11.	12.	13.	14.	15.
48	56	60	26	34
<u>25</u>	<u>25</u>	<u>25</u>	<u>25</u>	<u>25</u>
16.	17.	18.	19.	20.
38	42	58	62	64
<u>25</u>	<u>25</u>	<u>25</u>	<u>25</u>	<u>25</u>
21.	22.	23.	24.	25.
68	70	74	76	78
<u>25</u>	<u>25</u>	<u>25</u>	<u>25</u>	<u>25</u>

26. How much will 42 army rifles cost at \$ 25 apiece?

27. Find the cost of 3 dozen overcoats at \$ 25 each.

28. What will be the bill for 76 hours of work at 25 cents per hour?

29. What will it cost to fill an 18-gallon gasoline tank at 25 cents per gallon?

30. Find the cost of 8 cans of salmon eggs at 25 cents per can.

31. What will 12 pairs of men's hose cost at 25 cents a pair?

32. What will be the cost of 22 saucepans at 25 cents apiece?

33. Find the cost of 24 cans of sardines at 25 cents a can.

34. How much will 11 bottles of sewing machine oil cost at 25 cents a bottle?

35. Find the cost of three and a half dozen electric light bulbs at 25 cents each.

Exercise

By the same method find the results of the following indicated multiplications :

1. 122 <u>25</u>	2. 142 <u>25</u>	3. 145 <u>25</u>	4. 148 <u>25</u>
5. 150 <u>25</u>	6. 152 <u>25</u>	7. 202 <u>25</u>	8. 214 <u>25</u>
9. 242 <u>25</u>	10. 259 <u>25</u>	11. 261 <u>25</u>	12. 276 <u>25</u>
13. 291 <u>25</u>	14. 303 <u>25</u>	15. 313 <u>25</u>	16. 333 <u>25</u>
17. 345 <u>25</u>	18. 357 <u>25</u>	19. 390 <u>25</u>	20. 410 <u>25</u>
21. 516 <u>25</u>	22. 538 <u>25</u>	23. 568 <u>25</u>	24. 620 <u>25</u>
25. 649 <u>25</u>	26. 656 <u>25</u>	27. 659 <u>25</u>	28. 738 <u>25</u>

Multiplication by 125. Rule. — To multiply by 125, add three ciphers and divide by 8.

EXAMPLE. Multiply 72 by 125.

$$72 \times 125 = 72 \times 1000 \div 8 = 9000.$$

Exercise

By this method find the results of the following indicated multiplications :

1. 8 <u>125</u>	2. 12 <u>125</u>	3. 28 <u>125</u>	4. 24 <u>125</u>	5. 16 <u>125</u>
6. 48 <u>125</u>	7. 32 <u>125</u>	8. 40 <u>125</u>	9. 44 <u>125</u>	10. 56 <u>125</u>
11. 60 <u>125</u>	12. 68 <u>125</u>	13. 70 <u>125</u>	14. 76 <u>125</u>	15. 78 <u>125</u>
16. 74 <u>125</u>	17. 82 <u>125</u>	18. 84 <u>125</u>	19. 86 <u>125</u>	20. 88 <u>125</u>
21. 94 <u>125</u>	22. 112 <u>125</u>	23. 212 <u>125</u>	24. 438 <u>125</u>	25. 576 <u>125</u>

26. What will 500 copies of Hughes' "Community Civics" cost at \$ 1.25 per copy ?

27. What will 350 copies of Greer's "Textbook of Cooking" cost at \$ 1.25 per copy ?

28. Find the cost of 30 boxes of cartridges at \$ 1.25 a box.

29. What will 12 baseballs cost at a dollar and a quarter apiece ?

30. What will 50 pairs of tennis shoes cost at \$ 1.25 a pair ?

31. Find the cost of 16 phonograph records at \$ 1.25 each.

32. Find the cost of 6 dozen bandanna handkerchiefs at \$ 1.25 a dozen.
33. What will one dozen baseball mitts cost at \$ 2.50 each?
34. What will 125 sheep cost at \$ 12.50 per sheep?



AN OHIO PASTURE

To Multiply Such Numbers as 63 and 67. — When two numbers are such that the left-hand digits are the same and the sum of the right-hand digits is ten, multiplication is readily effected by the following rule.

Rule. — Increase the left-hand digit of one of the numbers by 1 and multiply it by the left-hand digit of the other number; this will give the first figure or figures in the answer. Multiply the two right-hand digits together; this will give the other figures in the answer.

EXAMPLE 1. Multiply 62 by 68.

$7 \times 6 = 42$, the first two figures in the answer ;

$2 \times 8 = 16$, the other two figures in the answer ;

therefore 62 times 68 is 4216.

EXAMPLE 2. Multiply 24 by 26.

$3 \times 2 = 6$, the first figure in the answer ;

$4 \times 6 = 24$, the other figures in the answer ;

therefore 24 times 26 is 624.

EXAMPLE 3. Multiply 69 by 61.

$7 \times 6 = 42$, the first figures in the answer ;

$9 \times 1 = 09$, the other figures in the answer ;

therefore 69 times 61 is 4209.

ORAL DRILL

Using the above rule find the following indicated products :

1.	2.	3.	4.	5.	6.
24	25	27	28	32	33
<u>26</u>	<u>25</u>	<u>23</u>	<u>22</u>	<u>38</u>	<u>37</u>
7.	8.	9.	10.	11.	12.
35	36	38	48	51	53
<u>35</u>	<u>34</u>	<u>32</u>	<u>42</u>	<u>59</u>	<u>57</u>
13.	14.	15.	16.	17.	18.
54	55	58	61	62	65
<u>56</u>	<u>55</u>	<u>52</u>	<u>69</u>	<u>68</u>	<u>65</u>
19.	20.	21.	22.	23.	24.
67	74	73	78	81	82
<u>63</u>	<u>76</u>	<u>77</u>	<u>72</u>	<u>89</u>	<u>88</u>

To Multiply by 11. — One or the other of the following cases can be used.

CASE I. — When the number consists of two figures whose sum is less than 10.

Rule. — Write their sum between them.

EXAMPLE 1. $32 \times 11 = 352$.

EXAMPLE 2. $54 \times 11 = 594$.

ORAL DRILL

Using the above rule find the following indicated products :

1.	2.	3.	4.	5.	6.
15	17	27	43	26	44
<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>
7.	8.	9.	10.	11.	12.
35	53	45	36	14	41
<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>
13.	14.	15.	16.	17.	18.
54	33	52	23	32	24
<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>
19.	20.	21.	22.	23.	24.
34	51	13	16	18	25
<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>

CASE II. — When the number consists of two figures whose sum is 10 or more.

Rule. — (1) Place the right-hand figure of the number as the right-hand figure of the answer.

(2) Add the figures of the number and place the right-hand figure of the sum in the tens' place.

(3) Add the other figure of this sum to the left-hand figure of the number for the left-hand figure or figures of the answer.

EXAMPLE 1. $76 \times 11 = 836$.

EXAMPLE 2. $87 \times 11 = 957$.

ORAL DRILL

Using the above rule find the following indicated products:

1.	2.	3.	4.	5.	6.
75	47	57	73	65	83
<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>
7.	8.	9.	10.	11.	12.
74	59	48	38	46	66
<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>
13.	14.	15.	16.	17.	18.
77	67	76	84	56	93
<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>
19.	20.	21.	22.	23.	24.
88	38	64	68	39	49
<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>

Division

Division is the inverse of multiplication and is the process of finding the number of times one number is contained in another.

In the statement "47 divided by 15 is $3\frac{2}{15}$," 47 is called the *dividend*, 15 is called the *divisor*, 3 is called the *quotient*, and 2 is the *remainder*.

EXAMPLE

$$\begin{array}{r}
 3 \text{ quotient} \\
 \text{divisor } 15 \overline{)47} \text{ dividend} \\
 \underline{45} \text{ product of quotient and dividend} \\
 2 \text{ remainder}
 \end{array}$$

Check. — Multiply the quotient by the divisor and add the remainder. The result is the dividend.

Divisibility of Numbers. — It is an advantage to be able to tell at a glance whether a certain number will be contained in another an exact number of times. The following tests of divisibility are some of the most useful ones.

1. All even numbers are divisible by 2.
2. A number is divisible by 3 if the sum of its digits is divisible by 3.
3. A number is divisible by 4 if the number represented by the right-hand two digits is zero or is divisible by 4.
4. A number is divisible by 5 if it ends with a 5 or a 0.
5. A number is divisible by 6 if it is an even number and the sum of its digits is divisible by 3.
6. A number is divisible by 8 if the number represented by the right-hand three digits is divisible by 8.
7. A number is divisible by 9 if the sum of its digits is divisible by 9.
8. A number is divisible by 10 if it ends with a 0.
9. A number is divisible by 12 if it is divisible by 3 and 4.
10. A number is divisible by 11 if the difference between the sums of the odd-placed and even-placed digits is zero or is divisible by 11.

ORAL DRILL

Perform the following indicated divisions :

1.	2.	3.	4.	5.
$8 \div 2$	$8 \div 3$	$8 \div 4$	$9 \div 2$	$12 \div 5$
6.	7.	8.	9.	10.
$12 \div 7$	$12 \div 9$	$14 \div 3$	$18 \div 7$	$22 \div 3$

11. $34 \div 2$	12. $36 \div 5$	13. $36 \div 11$	14. $45 \div 6$	15. $45 \div 8$
16. $52 \div 7$	17. $52 \div 8$	18. $56 \div 9$	19. $62 \div 8$	20. $64 \div 7$
21. $74 \div 8$	22. $75 \div 7$	23. $79 \div 11$	24. $84 \div 12$	25. $98 \div 9$

Long Division.—When all the steps in division are written down the process is called **Long Division**. Long division is generally used with divisors of two or more places.

EXAMPLE 1

$$\begin{array}{r}
 159\overset{3}{\underset{4}{6}} \\
 46 \overline{)7327} \\
 \underline{46} \\
 272 \\
 \underline{230} \\
 427 \\
 \underline{414} \\
 13
 \end{array}$$

Rule.—(1) Write the divisor at the left of the dividend.

(2) Find how many times the divisor is contained in the smallest group of figures on the left of the dividend, that will contain it, and write the quotient over the right-hand figure of this group.

(3) Multiply the divisor by this quotient and place the product under the group or partial dividend used.

(4) Subtract this product, and to the remainder annex the next figure of the dividend. Treat the number so formed as a new partial dividend.

(5) If any partial dividend will not contain the divisor, write a zero in the quotient, then annex the next figure of the dividend, and proceed as before.

(6) Divide as before, and continue the process until all the figures of the dividend have been used.

(7) If there is a remainder after the last division, write it with the divisor under it as part of the quotient.

Check.—Multiply the quotient by the divisor and add the remainder. The result is the dividend.

EXAMPLE 2

$$\begin{array}{r}
 304\overset{3}{\underset{8}{6}} \\
 68 \overline{)20675} \\
 \underline{204} \\
 275 \\
 \underline{272} \\
 3
 \end{array}$$

EXAMPLE 3

$$\begin{array}{r}
 37\overset{91}{\underset{342}{2}} \\
 342 \overline{)12745} \\
 \underline{1026} \\
 2485 \\
 \underline{2394} \\
 91
 \end{array}$$

Exercise

Perform the following indicated divisions :

1.	2.	3.	4.	5.
$624 \div 24$	$628 \div 25$	$631 \div 23$	$721 \div 28$	$734 \div 32$
6.	7.	8.	9.	10.
$745 \div 32$	$757 \div 36$	$782 \div 46$	$821 \div 54$	$832 \div 54$
11.	12.	13.	14.	15.
$922 \div 22$	$920 \div 34$	$933 \div 42$	$942 \div 45$	$965 \div 62$
16.	17.	18.	19.	20.
$1022 \div 65$	$1123 \div 68$	$1140 \div 68$	$1184 \div 68$	$1488 \div 72$
21.	22.	23.	24.	25.
$1728 \div 84$	$1610 \div 92$	$2212 \div 96$	$2728 \div 104$	$3633 \div 112$

Common Divisors. — When several numbers are to be divided by the same number, it is profitable to make a table of multiples of the divisor.

If, for example, 8476, 4093, 6298, 7647, 8643, 8786, etc., are all to be divided by 26, the student should make a table of multiples of 26, thus :

26	for by referring to the table of	EXAMPLES	
1 26	multiples it is readily seen each time what the next figure in the quotient will be.	326	$157\frac{11}{26}$
2 52		$26 \overline{)8476}$	$26 \overline{)4093}$
3 78		78	26
4 104		67	149
5 130		52	130
6 156		156	193
7 182		156	182
8 208			11
9 234			

Exercise

1. Divide each of the following numbers by 32 : 4608, 5698, 6336, 4678.

2. Divide each of the following numbers by 56 : 2016, 2912, 3864, 4564.

3. Divide each of the following numbers by 64 : 4224, 5888, 6786, 9075.

4. Divide each of the following numbers by 72 : 5017, 6824, 9248, 9428.

5. Divide each of the following numbers by 84 : 6234, 7280, 8294, 8762.

6. The normal monthly and annual temperature and precipitation, as published in the official records of the United States Weather Bureau, collected by the volunteer station at Redding, California, is as follows :

MONTHS	TEMPERATURE Degrees F.	PRECIPITATION Inches
January	45.2	7.29
February	49.2	4.22
March	53.8	4.75
April	60.2	3.01
May	67.2	2.19
June	75.6	.79
July	82.3	.09
August	81.0	.09
September	74.0	.67
October	64.5	2.48
November	54.1	3.79
December	<u>47.0</u>	<u>6.74</u>
Year		

Find the average or mean temperature, and the total yearly rainfall.

7. A New York Central train made a record run Sept. 11, 1895, from New York to Buffalo, $436\frac{1}{2}$ miles in 407 minutes. What was the average speed per hour?

8. The oil-burner *Santa Cruz* in February–March, 1914, went from New York to San Francisco via the Straits of Magellan, a distance of 13,000 miles, in 47 days and 4 hours. What was her average rate per hour?

Review Exercise

1. Add 4362, 4761, 5847, 6842, 4687, 3291, 4076, 5124, 4822.

2. Add \$ 57.60, \$ 34.20, \$ 16.15, \$ 1.75, \$ 4.20, \$ 16.00, \$ 42.50, \$ 62.10, \$ 4.85, \$ 5.40.

3. From the sum of all the numbers greater than 500 in the following list, take the sum of all the numbers which are less than 500 :

625, 732, 241, 357, 401, 562, 648, 321, 627, 438, 242, 742, 816.

4. A farm was purchased for \$ 6525 ; a barn was built on it at a cost of \$ 1880 ; a house for \$ 2135 ; improvements were made to the extent of \$ 875. What was the total outlay ?

5. If a man drives his automobile at an average rate of 22 miles an hour for 8 hours a day, how far does he drive during June ?

6. When the "Scott Special" left Los Angeles, Calif., July 9, 1905, and arrived in Chicago (2415.5 miles) July 11, having made the run in 44 hours, 54 minutes, what was the average speed ?

7. A man pays \$ 30 a month for rent and \$ 18 a year for water. How much does he pay out for rent and water in 10 years ?

8. The *Mauretania* in Sept., 1910, went from Queenstown to New York, 2181 nautical miles or knots, in 4 days, 10 hours, 41 minutes. What was her average speed in knots per hour ?

9. If 160 acres of land are sold for \$ 1000, what is the price per acre ?

10. On Nov. 19, 1916, Ruth Law made a long-distance non-stop aëroplane record flight from Chicago to Hornell, N. Y., 590 miles, in 5 hours, 45 minutes. What was her average speed per hour?

11. When a New York Central train made a record run Jan. 1, 1903, from Albany to Buffalo, 302 miles, in 295 minutes, what was the average speed per hour?



TWENTIETH CENTURY LIMITED GOING SIXTY MILES AN HOUR

12. On June 15, 1919, Captain John Alcock and Lieut. Arthur Brown flew from St. Johns, Newfoundland, to Clifden, Ireland, a distance of 1900 miles in 16 hours, 12 minutes. What was their average speed per hour?

13. A farm of 600 acres is planted as follows: fruit trees, 210 acres; grain, 135 acres; pasture, 60 acres; berries, 40 acres; artichokes, 54 acres. The remainder is taken up by farm buildings and yard. How much is occupied by buildings and yard?

14. If ten books occupy on an average one foot of shelf room, how many shelves four feet long will it require to hold 250 books?

15. If there are on an average nine words to a line and 26 lines to a page, how many pages will be occupied by a 100,000-word novel?

16. A boy pays \$18.75 for a gun, \$3.00 for ammunition, \$6.25 for boots, \$2.60 for a hunting-coat, \$2.25 for a knife, and \$1.00 for a hunting license. What is his total expenditure?



UNITED STATES WARSHIPS

17. The storage charge on certain goods is \$2.25 a month. What do the charges amount to in two years and a half?

18. 2250 books are to be boxed. If the boxes will hold on an average 120 books each, how many boxes are necessary?

19. In the course of a day a street car made 1000 stops. At each stop the car was stationary, on an average, five seconds. How many hours, all told, were spent in stops?

20. Find the totals in the following list of the naval forces of the United States in 1917 :

TYPE	NUMBER	TONS
Battleships (dreadnaught type)	17	467,450
Battleships (pre-dreadnaughts)	25	340,146
Small battleships and monitors	4	12,000
Battle cruisers	None	
Armored cruisers	10	140,080
Cruisers	15	75,629
Destroyers	74	60,219
Torpedo boats	6	1,082
Submarines	75	
Totals		

21. Find the totals in the following list showing the extent of a recent Naval Appropriation Bill :

TYPE	NUMBER	COST
Battleships	10	\$ 187,134,310
Battle cruisers	6	131,601,186
Scout cruisers	10	69,719,000
Destroyers	50	76,265,700
Fleet submarines	9	15,169,707
Coast submarines :		
800-ton type	3	3,918,906
Smaller type	55	44,346,610
Neff submarine	1	250,000
Fuel ships	3	4,794,816
Ammunition ships	2	4,929,300
Hospital ship	1	2,350,000
Repair ship	1	2,714,650
Transport	1	2,663,299
Destroyer tenders	2	4,395,884
Submarine tender	1	1,835,523
Gunboat	1	1,827,222
Totals		\$

CHAPTER III

FRACTIONS AND FACTORS

A Fraction is an expression denoting one or more equal parts of a unit. It is an indicated division, usually written in the form $\frac{2}{3}$.

The number below the line is called the *denominator*. It shows into how many parts the whole number has been divided.

The number above the line is called the *numerator*. It shows how many parts have been taken to form the fraction.

The numerator and denominator are called the *terms* of the fraction.

A *proper fraction* is one whose numerator is less than its denominator. Thus, $\frac{3}{4}$, $\frac{2}{3}$, and $\frac{1}{64}$ are proper fractions.

An *improper fraction* is one whose numerator is equal to, or greater than, its denominator. Thus, $\frac{4}{4}$, $\frac{5}{3}$, and $\frac{22}{7}$ are improper fractions.

A *mixed number* is a whole number with a fraction. Thus, $1\frac{2}{3}$, $2\frac{2}{5}$, and $3\frac{1}{7}$ are mixed numbers.

Mixed Numbers Expressed as Fractions

Rule. — To express a mixed number as a fraction, multiply the whole number by the denominator, add the numerator, and write the result over the denominator.

EXAMPLE. Express $3\frac{5}{8}$ as a fraction.

$8 \times 3 + 5 = 29$, the numerator of the fraction required.

The denominator is 8.

Then $3\frac{5}{8} = \frac{29}{8}$.

ORAL DRILL

Express as fractions the following mixed numbers :

- | | | | | |
|-------------------|----------------------|--------------------|--------------------|----------------------|
| 1. $1\frac{1}{2}$ | 7. $1\frac{1}{16}$ | 13. $4\frac{1}{6}$ | 19. $6\frac{7}{8}$ | 25. $8\frac{1}{6}$ |
| 2. $1\frac{1}{3}$ | 8. $1\frac{7}{16}$ | 14. $5\frac{1}{2}$ | 20. $7\frac{7}{8}$ | 26. $9\frac{1}{2}$ |
| 3. $1\frac{1}{4}$ | 9. $1\frac{11}{16}$ | 15. $5\frac{1}{3}$ | 21. $8\frac{1}{2}$ | 27. $9\frac{3}{8}$ |
| 4. $1\frac{1}{5}$ | 10. $1\frac{13}{16}$ | 16. $5\frac{1}{4}$ | 22. $8\frac{1}{3}$ | 28. $9\frac{5}{8}$ |
| 5. $1\frac{1}{6}$ | 11. $4\frac{1}{8}$ | 17. $5\frac{3}{8}$ | 23. $8\frac{1}{4}$ | 29. $10\frac{7}{8}$ |
| 6. $1\frac{1}{8}$ | 12. $4\frac{1}{7}$ | 18. $5\frac{5}{8}$ | 24. $8\frac{1}{5}$ | 30. $10\frac{7}{16}$ |

Exercise

Express as fractions the following mixed numbers :

- | | | | | |
|--------------------|---------------------|---------------------|---------------------|---------------------|
| 1. $47\frac{3}{5}$ | 7. $74\frac{3}{8}$ | 13. $97\frac{3}{4}$ | 19. $73\frac{2}{3}$ | 25. $92\frac{5}{7}$ |
| 2. $68\frac{4}{7}$ | 8. $59\frac{2}{3}$ | 14. $70\frac{5}{6}$ | 20. $87\frac{3}{4}$ | 26. $94\frac{2}{3}$ |
| 3. $96\frac{3}{4}$ | 9. $76\frac{4}{9}$ | 15. $27\frac{5}{8}$ | 21. $82\frac{5}{6}$ | 27. $69\frac{2}{3}$ |
| 4. $78\frac{2}{3}$ | 10. $38\frac{3}{8}$ | 16. $49\frac{1}{5}$ | 22. $80\frac{1}{5}$ | 28. $53\frac{3}{4}$ |
| 5. $56\frac{3}{4}$ | 11. $82\frac{3}{7}$ | 17. $96\frac{3}{4}$ | 23. $96\frac{5}{6}$ | 29. $57\frac{7}{8}$ |
| 6. $64\frac{5}{6}$ | 12. $65\frac{2}{5}$ | 18. $80\frac{4}{5}$ | 24. $73\frac{8}{9}$ | 30. $66\frac{7}{8}$ |

Improper Fractions Expressed as Mixed Numbers

Rule. — To express an improper fraction as a whole or mixed number, divide the numerator by the denominator.

EXAMPLE. Express $\frac{22}{7}$ as a whole or mixed number.

$$\begin{array}{r} 3\frac{1}{7} \\ 7 \overline{)22} \end{array}$$

ORAL DRILL

Express as whole or mixed numbers the following fractions :

- | | | | | |
|--------------------|---------------------|---------------------|---------------------|---------------------|
| 1. $\frac{20}{2}$ | 7. $\frac{20}{14}$ | 13. $\frac{30}{4}$ | 19. $\frac{30}{18}$ | 25. $\frac{36}{10}$ |
| 2. $\frac{20}{3}$ | 8. $\frac{20}{15}$ | 14. $\frac{30}{6}$ | 20. $\frac{30}{25}$ | 26. $\frac{36}{14}$ |
| 3. $\frac{20}{6}$ | 9. $\frac{20}{16}$ | 15. $\frac{30}{8}$ | 21. $\frac{36}{2}$ | 27. $\frac{36}{15}$ |
| 4. $\frac{20}{8}$ | 10. $\frac{20}{18}$ | 16. $\frac{30}{12}$ | 22. $\frac{36}{4}$ | 28. $\frac{36}{16}$ |
| 5. $\frac{20}{9}$ | 11. $\frac{30}{2}$ | 17. $\frac{30}{13}$ | 23. $\frac{36}{5}$ | 29. $\frac{36}{18}$ |
| 6. $\frac{20}{12}$ | 12. $\frac{30}{3}$ | 18. $\frac{30}{16}$ | 24. $\frac{36}{7}$ | 30. $\frac{36}{24}$ |

Exercise

Express as whole or mixed numbers the following fractions :

- | | | | | |
|---------------------|----------------------|----------------------|----------------------|----------------------|
| 1. $\frac{128}{12}$ | 7. $\frac{128}{32}$ | 13. $\frac{256}{30}$ | 19. $\frac{256}{75}$ | 25. $\frac{360}{38}$ |
| 2. $\frac{128}{14}$ | 8. $\frac{128}{56}$ | 14. $\frac{256}{32}$ | 20. $\frac{256}{85}$ | 26. $\frac{360}{45}$ |
| 3. $\frac{128}{16}$ | 9. $\frac{128}{64}$ | 15. $\frac{256}{35}$ | 21. $\frac{360}{12}$ | 27. $\frac{360}{52}$ |
| 4. $\frac{128}{22}$ | 10. $\frac{128}{84}$ | 16. $\frac{256}{36}$ | 22. $\frac{360}{15}$ | 28. $\frac{360}{75}$ |
| 5. $\frac{128}{24}$ | 11. $\frac{256}{24}$ | 17. $\frac{256}{45}$ | 23. $\frac{360}{32}$ | 29. $\frac{360}{85}$ |
| 6. $\frac{128}{25}$ | 12. $\frac{256}{25}$ | 18. $\frac{256}{64}$ | 24. $\frac{360}{35}$ | 30. $\frac{360}{95}$ |

Greatest Common Divisor

The **Greatest Common Divisor** of two or more numbers is the greatest number that is exactly contained in all of them. It is also called the *highest common factor*.

Thus, 12 is the greatest common divisor of 24 and 36.

Greatest Common Divisor is commonly abbreviated G. C. D. or H. C. F.

The principal use of greatest common divisor is its application in reducing fractions to their lowest terms.

Prime Numbers. — If numbers have no common factor other than 1, they are said to be *prime* to each other.

The greatest common divisor of two or more numbers can frequently be found by inspection, but when this is impossible, observe the following rule.

Rule.—(1) Separate the numbers into prime factors, and (2) find the product of the prime factors common to the numbers.

EXAMPLE. Find the G. C. D. of 54, 72, and 108.

$$\begin{array}{r}
 54 = 2 \times 3 \times 3 \times 3 \\
 72 = 2 \times 2 \times 2 \times 3 \times 3 \\
 108 = 2 \times 2 \times 3 \times 3 \times 3 \\
 \hline
 \text{G.C.D.} = 2 \times 3 \times 3 = 18
 \end{array}$$

Exercise

Find the G. C. D. of the following :

- | | | |
|---------------|-----------------|------------------|
| 1. 4, 16 | 11. 10, 35, 85 | 21. 32, 48, 128 |
| 2. 12, 78 | 12. 14, 21, 77 | 22. 51, 63, 132 |
| 3. 15, 75 | 13. 18, 72, 81 | 23. 36, 84, 132 |
| 4. 17, 51 | 14. 24, 60, 96 | 24. 60, 144, 576 |
| 5. 19, 171 | 15. 32, 48, 96 | 25. 38, 190, 760 |
| 6. 21, 105 | 16. 36, 54, 90 | 26. 32, 160, 808 |
| 7. 23, 207 | 17. 39, 52, 91 | 27. 33, 132, 143 |
| 8. 6, 8, 42 | 18. 63, 84, 126 | 28. 50, 125, 250 |
| 9. 8, 28, 84 | 19. 33, 77, 121 | 29. 64, 128, 256 |
| 10. 9, 33, 81 | 20. 30, 60, 250 | 30. 64, 160, 640 |

Factors

A Factor of a number is a number which is exactly contained in that number. Thus, 2, 3, 4, and 6 are all factors of 12.

In finding factors common to two or more numbers the following principles are of service.

1. A factor of a number is a factor of any multiple of that number.

2. A common factor of two numbers is a factor of the sum or difference of any of their multiples.

The greatest common divisor of even very large numbers can sometimes be readily found by the application of the principles just stated.

Fractions Reduced to Lowest Terms

A fraction is said to be in its *lowest terms* when its numerator and denominator contain no common factor other than 1. Thus, $\frac{2}{3}$ and $\frac{3}{7}$ are in their lowest terms, but $\frac{12}{18}$ is not, as its numerator and denominator contain a common factor 6.

Rule. — To reduce a fraction to its lowest terms, divide numerator and denominator by their common factors.

EXAMPLE. Reduce to lowest terms $\frac{36}{84}$.

$$\frac{36}{84} = \frac{12 \times 3}{12 \times 7} = \frac{3}{7}.$$

ORAL DRILL

Reduce to lowest terms the following fractions :

1. $\frac{12}{24}$

6. $\frac{6}{21}$

11. $\frac{14}{21}$

2. $\frac{8}{24}$

7. $\frac{8}{21}$

12. $\frac{16}{24}$

3. $\frac{9}{24}$

8. $\frac{9}{24}$

13. $\frac{24}{36}$

4. $\frac{21}{42}$

9. $\frac{11}{33}$

14. $\frac{18}{27}$

5. $\frac{36}{108}$

10. $\frac{12}{16}$

15. $\frac{18}{22}$

16. $\frac{20}{32}$	21. $\frac{15}{25}$	26. $\frac{35}{100}$
17. $\frac{22}{42}$	22. $\frac{15}{45}$	27. $\frac{5}{105}$
18. $\frac{36}{72}$	23. $\frac{15}{20}$	28. $\frac{7}{105}$
19. $\frac{72}{144}$	24. $\frac{35}{60}$	29. $\frac{14}{210}$
20. $\frac{6}{21}$	25. $\frac{45}{100}$	30. $\frac{15}{420}$

Exercise

Réduire to lowest terms the following fractions :

1. $\frac{96}{192}$	11. $\frac{96}{288}$	21. $\frac{140}{196}$
2. $\frac{120}{192}$	12. $\frac{108}{288}$	22. $\frac{84}{196}$
3. $\frac{128}{192}$	13. $\frac{128}{288}$	23. $\frac{64}{256}$
4. $\frac{144}{192}$	14. $\frac{144}{288}$	24. $\frac{96}{240}$
5. $\frac{160}{192}$	15. $\frac{160}{288}$	25. $\frac{84}{140}$
6. $\frac{168}{192}$	16. $\frac{180}{288}$	26. $\frac{128}{224}$
7. $\frac{32}{288}$	17. $\frac{192}{288}$	27. $\frac{180}{324}$
8. $\frac{36}{288}$	18. $\frac{216}{288}$	28. $\frac{216}{252}$
9. $\frac{64}{288}$	19. $\frac{56}{196}$	29. $\frac{160}{220}$
10. $\frac{72}{288}$	20. $\frac{112}{196}$	30. $\frac{212}{256}$

Least Common Multiple

The Least Common Multiple of two or more numbers is the smallest number which will contain them all without a remainder. It is usually abbreviated L. C. M.

24 is the least common multiple of 3, 8, and 12, because 24 is the smallest number which is divisible by all of the given numbers without a remainder.

Considerable use is made of least common multiple in the addition and subtraction of fractions, but the work is generally simple. Much of the labor in finding the least common multiple can be avoided by determining

whether some of the numbers given are factors of any of the others.

Rule. — (1) Separate each number into its prime factors.

(2) Find the product of these factors, using each factor the greatest number of times it occurs in any one of the given numbers.

EXAMPLE. Find the least common multiple of 2, 7, 3, 14, 15, and 30. As 30 is divisible by 15, the 15 need not be any longer considered, for any number that is divisible by 30 is also divisible by 15. In like manner the 2 and 3 can be disposed of, for they also are contained in 30. Similarly 7 is a divisor of 14 and hence 7 need not be further considered. The problem has now resolved itself into finding the least common multiple of 14 and 30. Either of the following arrangements can be used.

$$\begin{array}{rcl}
 14 & = & 2 \times 7 \\
 30 & = & 2 \times 3 \times 5 \\
 \hline
 \text{L. C. M.} & = & 2 \times 7 \times 3 \times 5 = 210
 \end{array}
 \qquad
 \begin{array}{r}
 2 \overline{)14 \quad 30} \\
 \underline{7 \quad 15}
 \end{array}
 \qquad
 \text{L. C. M.} = 2 \times 7 \times 15 = 210$$

ORAL DRILL

Find the Least Common Multiple of the numbers given in each of the following examples :

- | | | |
|--------------|--------------|-------------------|
| 1. 2, 3, 5 | 11. 3, 9, 15 | 21. 44, 8, 88 |
| 2. 3, 4, 5 | 12. 3, 9, 45 | 22. 11, 121, 33 |
| 3. 3, 5, 9 | 13. 3, 9, 54 | 23. 4, 60, 20 |
| 4. 4, 6, 12 | 14. 3, 9, 63 | 24. 8, 72, 3 |
| 5. 4, 6, 16 | 15. 4, 5, 6 | 25. 8, 56, 16 |
| 6. 4, 6, 20 | 16. 12, 16 | 26. 11, 22, 33 |
| 7. 6, 8, 12 | 17. 40, 50 | 27. 24, 36, 72 |
| 8. 6, 8, 16 | 18. 50, 210 | 28. 24, 72, 96 |
| 9. 6, 8, 20 | 19. 75, 300 | 29. 24, 72, 240 |
| 10. 6, 8, 24 | 20. 21, 35 | 30. 12, 144, 1728 |

Exercise

Find the Least Common Multiple of the numbers given in each of the following examples :

- | | | |
|-----------------|-----------------|-------------------|
| 1. 16, 24, 40 | 11. 18, 54, 90 | 21. 88, 16, 176 |
| 2. 24, 32, 40 | 12. 18, 54, 270 | 22. 22, 242, 66 |
| 3. 24, 40, 72 | 13. 18, 54, 324 | 23. 8, 120, 40 |
| 4. 32, 48, 96 | 14. 18, 54, 378 | 24. 16, 144, 6 |
| 5. 32, 48, 128 | 15. 24, 30, 36 | 25. 16, 112, 32 |
| 6. 32, 48, 160 | 16. 72, 96 | 26. 22, 44, 66 |
| 7. 48, 64, 96 | 17. 240, 300 | 27. 48, 72, 144 |
| 8. 48, 64, 128 | 18. 300, 1260 | 28. 48, 144, 192 |
| 9. 48, 64, 160 | 19. 450, 1800 | 29. 48, 144, 480 |
| 10. 48, 64, 192 | 20. 126, 210 | 30. 24, 288, 3456 |

Addition of Fractions

The fundamental principle in adding fractions is that the denominators must be equal.

Common Denominator. — If two or more fractions have the same denominator they are said to have a *common denominator*. Thus, $\frac{5}{12}$ and $\frac{7}{12}$ have the common denominator 12.

Similar Fractions. — Fractions that have a common denominator are called *similar fractions*.

Rule. — To add fractions, (1) reduce them to similar fractions, (2) write the sum of the numerators over the least common denominator, and (3) reduce the result to lowest terms.

To add mixed numbers, add separately the whole numbers and the fractions and find the sum of the results.

EXAMPLE. Add $2\frac{2}{3}$, $3\frac{1}{2}$, $4\frac{1}{6}$.

$$2\frac{2}{3} = 2\frac{4}{6}$$

$$3\frac{1}{2} = 3\frac{3}{6}$$

$$4\frac{1}{6} = 4\frac{1}{6}$$

$$9\frac{8}{6} = 10\frac{2}{6} \text{ or } 10\frac{1}{3}, \text{ the sum.}$$

Exercise

In each of the following examples find the sum of the numbers given.

1. $\frac{3}{12}, \frac{5}{12}, \frac{7}{12}, \frac{11}{12}$

2. $\frac{5}{17}, \frac{6}{17}, \frac{9}{17}, \frac{11}{17}, \frac{14}{17}, \frac{16}{17}$

3. $\frac{4}{28}, \frac{9}{28}, \frac{11}{28}, \frac{13}{28}, \frac{13}{28}, \frac{19}{28}$

4. $\frac{11}{144}, \frac{19}{144}, \frac{29}{144}, \frac{107}{144}$

5. $\frac{1}{4}, \frac{7}{8}, \frac{11}{12}, \frac{5}{9}$

6. $\frac{7}{11}, \frac{9}{22}, 1\frac{2}{3}, \frac{1}{6}$

7. $\frac{5}{17}, \frac{3}{34}, \frac{2}{17}, \frac{1}{2}$

8. $2\frac{14}{15}, \frac{17}{20}, \frac{19}{30}, 3\frac{9}{10}$

9. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}$

10. $1\frac{3}{4}, 2\frac{5}{6}, 4\frac{3}{4}$

11. $3\frac{4}{5}, 7\frac{1}{2}, 8\frac{3}{4}$

12. $16\frac{2}{8}, 8\frac{7}{8}, 9\frac{3}{5}, 3\frac{1}{4}, 1\frac{7}{8}$

13. $1\frac{1}{10}, 2\frac{2}{5}, 5\frac{1}{5}$

14. $2\frac{1}{16}, 3\frac{3}{8}, 4\frac{1}{4}$

15. $3\frac{3}{16}, 3\frac{3}{8}, 3\frac{3}{4}$

16. $5\frac{1}{6}, 4\frac{1}{3}, 3\frac{1}{2}$

Prime Denominators. — When the denominators of two or more fractions are prime to each other observe the following rule.

Rule. — (1) Multiply the denominator of each fraction by the numerator of the other, (2) add these products, and (3) write it over the product of the denominators.

EXAMPLE. Add $\frac{2}{3}$ and $\frac{4}{5}$.

$$2 \times 5 = 10$$

$$3 \times 4 = 12$$

Their sum = 22, the numerator of the answer ;

$$5 \times 3 = 15, \text{ the denominator of the answer.}$$

Then $\frac{2}{3}$ plus $\frac{4}{5}$ is $\frac{22}{15}$, or $1\frac{7}{15}$.

ORAL DRILL

In each of the following examples, find the sum of the numbers given.

- | | | |
|--------------------------------|--------------------------------|--------------------------------|
| 1. $\frac{2}{3}, \frac{3}{4}$ | 11. $\frac{3}{4}, \frac{1}{3}$ | 21. $\frac{5}{8}, \frac{1}{5}$ |
| 2. $\frac{2}{3}, \frac{2}{5}$ | 12. $\frac{3}{4}, \frac{2}{3}$ | 22. $\frac{5}{8}, \frac{2}{5}$ |
| 3. $\frac{2}{3}, \frac{3}{5}$ | 13. $\frac{3}{4}, \frac{1}{5}$ | 23. $\frac{5}{8}, \frac{3}{5}$ |
| 4. $\frac{2}{3}, \frac{1}{5}$ | 14. $\frac{3}{4}, \frac{2}{5}$ | 24. $\frac{5}{8}, \frac{4}{5}$ |
| 5. $\frac{2}{3}, \frac{4}{5}$ | 15. $\frac{3}{4}, \frac{3}{5}$ | 25. $\frac{5}{8}, \frac{1}{3}$ |
| 6. $\frac{2}{3}, \frac{1}{8}$ | 16. $\frac{3}{4}, \frac{4}{5}$ | 26. $\frac{5}{8}, \frac{2}{3}$ |
| 7. $\frac{2}{3}, \frac{1}{4}$ | 17. $\frac{3}{8}, \frac{1}{5}$ | 27. $\frac{5}{8}, \frac{3}{4}$ |
| 8. $\frac{2}{3}, \frac{3}{8}$ | 18. $\frac{3}{8}, \frac{2}{5}$ | 28. $\frac{5}{8}, \frac{1}{6}$ |
| 9. $\frac{2}{3}, \frac{1}{2}$ | 19. $\frac{3}{8}, \frac{1}{5}$ | 29. $\frac{5}{8}, \frac{5}{6}$ |
| 10. $\frac{2}{3}, \frac{5}{8}$ | 20. $\frac{3}{8}, \frac{4}{5}$ | 30. $\frac{5}{8}, \frac{5}{9}$ |

Fractions Whose Numerators Are 1. — When two fractions are to be added where both numerators are 1, and the denominators are prime to each other, observe the following rule.

Rule. — (1) Add the denominators together for the numerator of the answer; (2) multiply the denominators together for the denominator of the answer.

EXAMPLE. Add $\frac{1}{4}$ and $\frac{1}{5}$.

$4 + 5 = 9$, the numerator of the answer;

$4 \times 5 = 20$, the denominator of the answer.

Then $\frac{1}{4} + \frac{1}{5} = \frac{9}{20}$, the sum.

ORAL DRILL

In each of the following examples find the sum of the numbers given.

- | | | |
|-------------------------------|-------------------------------|-------------------------------|
| 1. $\frac{1}{2}, \frac{1}{3}$ | 3. $\frac{1}{2}, \frac{1}{7}$ | 5. $\frac{1}{3}, \frac{1}{4}$ |
| 2. $\frac{1}{2}, \frac{1}{5}$ | 4. $\frac{1}{2}, \frac{1}{9}$ | 6. $\frac{1}{3}, \frac{1}{5}$ |

- | | | |
|---------------------------------|---------------------------------|----------------------------------|
| 7. $\frac{1}{3}, \frac{1}{7}$ | 15. $\frac{1}{5}, \frac{1}{6}$ | 23. $\frac{1}{6}, \frac{1}{13}$ |
| 8. $\frac{1}{3}, \frac{1}{8}$ | 16. $\frac{1}{5}, \frac{1}{7}$ | 24. $\frac{1}{7}, \frac{1}{8}$ |
| 9. $\frac{1}{3}, \frac{1}{10}$ | 17. $\frac{1}{5}, \frac{1}{8}$ | 25. $\frac{1}{7}, \frac{1}{9}$ |
| 10. $\frac{1}{3}, \frac{1}{11}$ | 18. $\frac{1}{5}, \frac{1}{9}$ | 26. $\frac{1}{7}, \frac{1}{10}$ |
| 11. $\frac{1}{4}, \frac{1}{5}$ | 19. $\frac{1}{5}, \frac{1}{12}$ | 27. $\frac{1}{7}, \frac{1}{12}$ |
| 12. $\frac{1}{4}, \frac{1}{7}$ | 20. $\frac{1}{5}, \frac{1}{14}$ | 28. $\frac{1}{8}, \frac{1}{9}$ |
| 13. $\frac{1}{4}, \frac{1}{9}$ | 21. $\frac{1}{6}, \frac{1}{7}$ | 29. $\frac{1}{8}, \frac{1}{15}$ |
| 14. $\frac{1}{4}, \frac{1}{11}$ | 22. $\frac{1}{6}, \frac{1}{11}$ | 30. $\frac{1}{11}, \frac{1}{12}$ |

Subtraction of Fractions

In the subtraction of fractions, as in their addition, the denominators must be made equal.

Rule. — To subtract one fraction from another, (1) reduce them to similar fractions having the least common denominator, (2) write the difference of the numerators over the least common denominator, and (3) reduce the result to lowest terms.

EXAMPLE. From $\frac{2}{3}$ take $\frac{1}{2}$.

$$\begin{aligned}\frac{2}{3} &= \frac{4}{6} \\ \frac{1}{2} &= \frac{3}{6} \\ &\frac{1}{6}\end{aligned}$$

Exercise

In each of the following examples, find the difference between the numbers given :

- | | | |
|-------------------------------|--------------------------------|--------------------------------|
| 1. $\frac{2}{3}, \frac{3}{4}$ | 6. $\frac{2}{3}, \frac{1}{8}$ | 11. $\frac{3}{4}, \frac{1}{3}$ |
| 2. $\frac{2}{3}, \frac{2}{5}$ | 7. $\frac{2}{3}, \frac{1}{4}$ | 12. $\frac{3}{4}, \frac{2}{3}$ |
| 3. $\frac{2}{3}, \frac{3}{5}$ | 8. $\frac{2}{3}, \frac{3}{8}$ | 13. $\frac{3}{4}, \frac{1}{5}$ |
| 4. $\frac{2}{3}, \frac{1}{5}$ | 9. $\frac{2}{3}, \frac{1}{2}$ | 14. $\frac{3}{4}, \frac{2}{5}$ |
| 5. $\frac{2}{3}, \frac{4}{5}$ | 10. $\frac{2}{3}, \frac{5}{8}$ | 15. $\frac{3}{4}, \frac{3}{5}$ |

16. $\frac{3}{4}, \frac{4}{5}$

17. $\frac{3}{8}, \frac{1}{5}$

18. $\frac{3}{8}, \frac{2}{5}$

19. $\frac{3}{8}, \frac{2}{5}$

20. $\frac{3}{8}, \frac{4}{5}$

21. $\frac{5}{8}, \frac{1}{5}$

22. $\frac{5}{8}, \frac{1}{5}$

23. $\frac{5}{8}, \frac{2}{5}$

24. $\frac{5}{8}, \frac{4}{5}$

25. $\frac{5}{8}, \frac{1}{3}$

26. $\frac{5}{8}, \frac{2}{3}$

27. $\frac{5}{8}, \frac{3}{4}$

28. $\frac{5}{8}, \frac{1}{6}$

29. $\frac{5}{8}, \frac{5}{6}$

30. $\frac{5}{8}, \frac{5}{9}$

Prime Denominators. — When the numerator of each fraction is 1, and the denominators are prime to each other, observe the following rule.

Rule. — (1) Find the difference of the denominators for the numerator of the answer; (2) multiply the denominators together for the denominator of the answer.

EXAMPLE. From $\frac{1}{4}$ subtract $\frac{1}{5}$.

$5 - 4 = 1$, the numerator of the answer;

$4 \times 5 = 20$, the denominator of the answer.

Then $\frac{1}{4} - \frac{1}{5} = \frac{1}{20}$, the difference.

ORAL DRILL

In each of the following examples find the difference between the numbers given.

1. $\frac{1}{2}, \frac{1}{3}$

7. $\frac{1}{3}, \frac{1}{7}$

13. $\frac{1}{4}, \frac{1}{9}$

19. $\frac{1}{5}, \frac{1}{12}$

25. $\frac{1}{7}, \frac{1}{9}$

2. $\frac{1}{2}, \frac{1}{5}$

8. $\frac{1}{3}, \frac{1}{8}$

14. $\frac{1}{4}, \frac{1}{11}$

20. $\frac{1}{5}, \frac{1}{14}$

26. $\frac{1}{7}, \frac{1}{10}$

3. $\frac{1}{2}, \frac{1}{7}$

9. $\frac{1}{3}, \frac{1}{10}$

15. $\frac{1}{5}, \frac{1}{6}$

21. $\frac{1}{6}, \frac{1}{7}$

27. $\frac{1}{7}, \frac{1}{12}$

4. $\frac{1}{2}, \frac{1}{9}$

10. $\frac{1}{3}, \frac{1}{11}$

16. $\frac{1}{5}, \frac{1}{7}$

22. $\frac{1}{6}, \frac{1}{11}$

28. $\frac{1}{8}, \frac{1}{9}$

5. $\frac{1}{3}, \frac{1}{4}$

11. $\frac{1}{4}, \frac{1}{5}$

17. $\frac{1}{5}, \frac{1}{8}$

23. $\frac{1}{6}, \frac{1}{13}$

29. $\frac{1}{8}, \frac{1}{15}$

6. $\frac{1}{3}, \frac{1}{5}$

12. $\frac{1}{4}, \frac{1}{7}$

18. $\frac{1}{5}, \frac{1}{9}$

24. $\frac{1}{7}, \frac{1}{8}$

30. $\frac{1}{11}, \frac{1}{12}$

Mixed Numbers. — When the numbers are mixed numbers and the fractional part of the subtrahend is greater than the fractional part of the minuend, observe the following rule.

Rule.—(1) Increase the fraction in the minuend by 1,
 (2) decrease the whole number by the same amount, and
 (3) proceed as before.

EXAMPLE. From $7\frac{2}{5}$ take $4\frac{3}{4}$.

$$\begin{aligned} 7\frac{2}{5} &= 7\frac{8}{20} = 6\frac{28}{20} \\ 4\frac{3}{4} &= 4\frac{15}{20} = \underline{4\frac{15}{20}} \\ &2\frac{13}{20}, \text{ the difference.} \end{aligned}$$

It will be observed in the fraction $\frac{8}{20}$ that if the numerator and denominator are added, the sum is the numerator of the fraction at the right.

Exercise

In each of the following examples, find the difference between the numbers given :

- | | | | |
|---------------------------------|-----------------------------------|------------------------------------|-------------------------------------|
| 1. $4\frac{2}{3}, 1\frac{3}{4}$ | 9. $6\frac{2}{3}, 2\frac{4}{5}$ | 17. $10\frac{1}{5}, 3\frac{3}{8}$ | 25. $12\frac{5}{8}, 8\frac{1}{3}$ |
| 2. $5\frac{2}{3}, 2\frac{2}{5}$ | 10. $8\frac{2}{3}, 4\frac{5}{6}$ | 18. $12\frac{2}{5}, 7\frac{3}{8}$ | 26. $16\frac{5}{8}, 11\frac{2}{3}$ |
| 3. $6\frac{2}{3}, 3\frac{3}{5}$ | 11. $8\frac{1}{3}, 2\frac{3}{4}$ | 19. $16\frac{3}{5}, 11\frac{3}{8}$ | 27. $13\frac{5}{8}, 10\frac{3}{4}$ |
| 4. $7\frac{2}{3}, 4\frac{1}{5}$ | 12. $10\frac{2}{3}, 4\frac{3}{4}$ | 20. $14\frac{4}{5}, 12\frac{3}{8}$ | 28. $15\frac{5}{8}, 10\frac{1}{6}$ |
| 5. $4\frac{2}{3}, 1\frac{4}{5}$ | 13. $12\frac{1}{5}, 6\frac{3}{4}$ | 21. $14\frac{5}{8}, 4\frac{1}{5}$ | 29. $12\frac{5}{8}, 8\frac{5}{6}$ |
| 6. $5\frac{2}{3}, 2\frac{1}{8}$ | 14. $9\frac{2}{5}, 1\frac{3}{4}$ | 22. $12\frac{5}{8}, 2\frac{2}{5}$ | 30. $18\frac{5}{8}, 12\frac{5}{9}$ |
| 7. $6\frac{2}{3}, 3\frac{1}{4}$ | 15. $14\frac{3}{5}, 6\frac{3}{4}$ | 23. $10\frac{5}{8}, 7\frac{3}{5}$ | 31. $24\frac{7}{16}, 17\frac{1}{8}$ |
| 8. $7\frac{2}{3}, 3\frac{3}{4}$ | 16. $11\frac{4}{5}, 3\frac{3}{4}$ | 24. $8\frac{5}{8}, 4\frac{4}{5}$ | 32. $32\frac{3}{32}, 7\frac{3}{4}$ |

Exercise

Using the best method in each case, find in the first fifteen of the following examples the difference between the numbers given, and in the last seventeen, the sum :

- | | | | |
|---------------------------------|---------------------------------|-----------------------------------|-----------------------------------|
| 1. $3\frac{1}{8}, 2\frac{1}{2}$ | 5. $5\frac{1}{4}, 2\frac{1}{3}$ | 9. $5\frac{1}{10}, 2\frac{1}{3}$ | 13. $4\frac{1}{9}, 2\frac{1}{4}$ |
| 2. $3\frac{1}{5}, 1\frac{1}{2}$ | 6. $4\frac{2}{5}, 1\frac{1}{3}$ | 10. $11\frac{4}{7}, 3\frac{1}{3}$ | 14. $5\frac{1}{11}, 2\frac{3}{4}$ |
| 3. $4\frac{1}{7}, 2\frac{1}{2}$ | 7. $3\frac{1}{7}, 1\frac{1}{3}$ | 11. $2\frac{1}{5}, 1\frac{1}{4}$ | 15. $6\frac{1}{6}, 3\frac{1}{5}$ |
| 4. $5\frac{2}{9}, 2\frac{1}{2}$ | 8. $4\frac{1}{8}, 3\frac{1}{3}$ | 12. $3\frac{1}{7}, 1\frac{3}{4}$ | 16. $7\frac{1}{7}, 3\frac{2}{5}$ |

- | | | | |
|-----------------------------------|------------------------------------|-----------------------------------|-------------------------------------|
| 17. $6\frac{3}{8}, 3\frac{1}{5}$ | 21. $6\frac{1}{7}, 2\frac{1}{6}$ | 25. $5\frac{4}{9}, 3\frac{2}{7}$ | 29. $14\frac{1}{15}, 8\frac{1}{8}$ |
| 18. $5\frac{1}{9}, 2\frac{1}{5}$ | 22. $5\frac{1}{11}, 2\frac{1}{6}$ | 26. $6\frac{1}{10}, 2\frac{1}{7}$ | 30. $12\frac{1}{12}, 9\frac{1}{11}$ |
| 19. $4\frac{7}{12}, 2\frac{1}{5}$ | 23. $4\frac{4}{13}, 1\frac{1}{16}$ | 27. $8\frac{1}{12}, 4\frac{1}{7}$ | 31. $15\frac{1}{16}, 11\frac{3}{8}$ |
| 20. $5\frac{1}{14}, 2\frac{1}{5}$ | 24. $4\frac{1}{8}, 2\frac{1}{7}$ | 28. $10\frac{1}{9}, 7\frac{1}{8}$ | 32. $44\frac{4}{5}, 33\frac{3}{8}$ |

Multiplication and Division of Fractions

In the multiplication and division of fractions it is well to keep in mind the following *Principles*:

1. Multiplying the numerator multiplies the fraction.
2. Multiplying the denominator divides the fraction.
3. Dividing the numerator divides the fraction.
4. Dividing the denominator multiplies the fraction.
5. Multiplying both numerator and denominator by the same number does not change the value of the fraction.
6. Dividing both numerator and denominator by the same number does not change the value of the fraction.

Multiplication of Fractions

Rule.—To multiply one fraction by another : (1) multiply the numerators together for the numerator of the answer ; (2) multiply the denominators together for the denominator of the answer.

EXAMPLE 1. Multiply $\frac{2}{3}$ by $\frac{4}{5}$.

$2 \times 4 = 8$, the numerator of the answer ;

$3 \times 5 = 15$, the denominator of the answer.

Then $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$.

When a fraction is to be multiplied by a whole number the whole number can be considered as a fraction whose denominator is 1.

EXAMPLE 2. Multiply $\frac{2}{3}$ by 5.

$$\frac{2}{3} \times 5 = \frac{2}{3} \times \frac{5}{1} = \frac{10}{3} = 3\frac{1}{3}.$$

If the numerator and denominator contain common factors, remove them in accordance with Principle 6 stated above. This process is called *cancellation*.

EXAMPLE 3. Multiply $\frac{2}{3}$ by $\frac{6}{7}$.

$$\frac{2}{\cancel{3}^1} \times \frac{\overset{2}{\cancel{6}}}{7} = \frac{4}{7}.$$

If several fractions are to be multiplied together, the process is the same.

EXAMPLE 4. Find the product of $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{3}{8}$.

$$\frac{\overset{1}{\cancel{2}}}{\cancel{3}^1} \times \frac{\overset{1}{\cancel{4}}}{5} \times \frac{\overset{1}{\cancel{3}}}{\cancel{8}^2} = \frac{1}{5}.$$

Observe that as all the factors (except the 1's) of the numerator are cancelled, the numerator of the answer is 1.

ORAL DRILL

In each of the following examples find the product of the numbers given:

- | | | | | |
|-------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|
| 1. $\frac{2}{3}, \frac{3}{4}$ | 7. $\frac{2}{3}, \frac{1}{4}$ | 13. $\frac{3}{4}, \frac{1}{5}$ | 19. $\frac{3}{8}, \frac{3}{5}$ | 25. $\frac{5}{8}, \frac{1}{3}$ |
| 2. $\frac{2}{3}, \frac{2}{5}$ | 8. $\frac{2}{3}, \frac{3}{8}$ | 14. $\frac{3}{4}, \frac{2}{5}$ | 20. $\frac{3}{8}, \frac{4}{5}$ | 26. $\frac{5}{8}, \frac{2}{3}$ |
| 3. $\frac{2}{3}, \frac{3}{5}$ | 9. $\frac{2}{3}, \frac{1}{2}$ | 15. $\frac{3}{4}, \frac{3}{5}$ | 21. $\frac{5}{8}, \frac{1}{5}$ | 27. $\frac{5}{8}, \frac{3}{4}$ |
| 4. $\frac{2}{3}, \frac{1}{5}$ | 10. $\frac{2}{3}, \frac{5}{8}$ | 16. $\frac{3}{4}, \frac{4}{5}$ | 22. $\frac{5}{8}, \frac{1}{5}$ | 28. $\frac{5}{8}, \frac{1}{6}$ |
| 5. $\frac{2}{3}, \frac{4}{5}$ | 11. $\frac{3}{4}, \frac{1}{3}$ | 17. $\frac{3}{8}, \frac{1}{5}$ | 23. $\frac{5}{8}, \frac{3}{5}$ | 29. $\frac{5}{8}, \frac{5}{6}$ |
| 6. $\frac{2}{3}, \frac{1}{8}$ | 12. $\frac{3}{4}, \frac{2}{3}$ | 18. $\frac{3}{8}, \frac{2}{5}$ | 24. $\frac{5}{8}, \frac{4}{5}$ | 30. $\frac{5}{8}, \frac{5}{9}$ |

Mixed Numbers. — To multiply a fraction by a mixed number, or to multiply one mixed number by another, observe the following rule.

Rule. — (1) Express the mixed numbers as fractions, and (2) proceed as before.

EXAMPLE 1. Multiply $\frac{3}{8}$ by $3\frac{1}{2}$.

$$\frac{3}{8} \times 3\frac{1}{2} = \frac{3}{8} \times \frac{7}{2} = \frac{21}{16} = 1\frac{5}{16}.$$

EXAMPLE 2. Multiply $1\frac{1}{2}$ by $4\frac{1}{3}$.

$$1\frac{1}{2} \times 4\frac{1}{3} = \frac{1}{2} \times \frac{13}{3} = \frac{13}{6} = 2\frac{1}{2}.$$

Exercise

In each of the following examples find the product of the numbers given:

- | | | | |
|----------------------------------|-----------------------------------|-------------------------------------|--|
| 1. $3\frac{1}{3}, 3$ | 9. $5\frac{1}{10}, 2\frac{4}{17}$ | 17. $5\frac{3}{5}, 5$ | 25. $2\frac{1}{3}, \frac{1}{2}, \frac{3}{7}$ |
| 2. $3\frac{1}{5}, 1\frac{1}{16}$ | 10. $5\frac{4}{7}, 3\frac{2}{13}$ | 18. $2\frac{5}{6}, 2\frac{5}{6}$ | 26. $4\frac{1}{2}, \frac{4}{9}, 36$ |
| 3. $4\frac{2}{7}, 2\frac{2}{15}$ | 11. $2\frac{1}{2}, 4$ | 19. $6, 3\frac{2}{3}$ | 27. $5, \frac{3}{8}, \frac{1}{15}$ |
| 4. $2\frac{2}{9}, 3\frac{3}{10}$ | 12. $4\frac{1}{2}, 2$ | 20. $8, 4\frac{3}{4}$ | 28. $7\frac{1}{2}, 2, 1\frac{1}{3}$ |
| 5. $5\frac{1}{4}, 4\frac{4}{7}$ | 13. $5\frac{1}{5}, 5$ | 21. $\frac{2}{3}, 3, \frac{1}{5}$ | 29. $3\frac{1}{3}, 2\frac{1}{2}, 12$ |
| 6. $1\frac{2}{9}, 2\frac{1}{11}$ | 14. $2\frac{1}{5}, \frac{5}{11}$ | 22. $\frac{2}{3}, \frac{3}{4}, 5$ | 30. $2\frac{1}{3}, 3\frac{1}{2}, 6$ |
| 7. $4\frac{1}{5}, 3\frac{2}{3}$ | 15. $3\frac{1}{6}, 6$ | 23. $\frac{1}{3}, 1\frac{1}{3}, 9$ | 31. $4\frac{1}{2}, 5\frac{1}{3}, 36$ |
| 8. $4\frac{1}{8}, 4\frac{3}{11}$ | 16. $4\frac{2}{5}, 4\frac{1}{11}$ | 24. $2\frac{1}{2}, 2\frac{1}{4}, 2$ | 32. $12\frac{1}{4}, 8\frac{1}{5}, 120$ |

Division of Fractions

Rule. — To divide one fraction by another, invert the divisor and proceed as in multiplication.

EXAMPLE 1. Divide $\frac{2}{3}$ by $\frac{8}{15}$.

$$\frac{2}{3} \div \frac{8}{15} = \frac{2}{3} \times \frac{15}{8} = \frac{5}{4} = 1\frac{1}{4}.$$

EXAMPLE 2. Divide $3\frac{3}{8}$ by $1\frac{1}{16}$.

$$3\frac{3}{8} = \frac{27}{8}; 1\frac{1}{16} = \frac{17}{16}.$$

$$\frac{27}{8} \div \frac{17}{16} = \frac{27}{8} \times \frac{16}{17} = 2.$$

EXAMPLE 3. Divide 10 by $3\frac{1}{3}$.

$$3\frac{1}{3} = \frac{10}{3}.$$

$$10 \div \frac{10}{3} = \frac{10}{1} \times \frac{3}{10} = 3.$$

Exercise

In each of the following examples divide the first number by the second one:

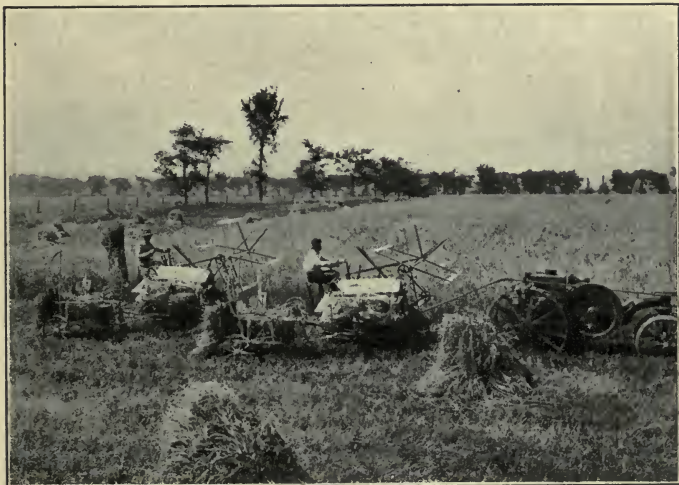
- | | | | |
|---------------------------------|-----------------------------------|----------------------------------|---------------------------|
| 1. $4\frac{2}{3}, 1\frac{3}{4}$ | 9. $9\frac{1}{3}, 2\frac{4}{5}$ | 17. $\frac{5}{16}, \frac{3}{8}$ | 25. 100, $8\frac{1}{3}$ |
| 2. $5\frac{2}{3}, 2\frac{2}{5}$ | 10. $8\frac{2}{3}, 4\frac{5}{6}$ | 18. $1\frac{1}{2}, \frac{7}{16}$ | 26. 100, $16\frac{2}{3}$ |
| 3. $6\frac{2}{3}, 3\frac{3}{5}$ | 11. $8\frac{1}{3}, \frac{1}{3}$ | 19. $4\frac{1}{2}, 5$ | 27. 100, $62\frac{1}{2}$ |
| 4. $7\frac{2}{3}, 4\frac{3}{5}$ | 12. $10\frac{2}{3}, 2\frac{2}{7}$ | 20. $4\frac{1}{2}, 6$ | 28. 1000, $1\frac{1}{4}$ |
| 5. $4\frac{2}{3}, 1\frac{2}{5}$ | 13. 12, $3\frac{1}{4}$ | 21. 14, $4\frac{1}{2}$ | 29. 1000, $12\frac{1}{2}$ |
| 6. $5\frac{2}{3}, 2\frac{1}{8}$ | 14. 12, $3\frac{1}{3}$ | 22. 10, $3\frac{1}{5}$ | 30. 1000, $37\frac{1}{2}$ |
| 7. $6\frac{2}{3}, 3\frac{3}{4}$ | 15. 20, $2\frac{1}{2}$ | 23. 10, $3\frac{2}{5}$ | 31. 1000, $62\frac{1}{2}$ |
| 8. $8\frac{1}{3}, 3\frac{3}{4}$ | 16. $\frac{5}{8}, \frac{3}{8}$ | 24. 100, $6\frac{1}{4}$ | 32. 1000, $87\frac{1}{2}$ |

Review Exercise

- What is $\frac{5}{8}$ of 25?
- What is $\frac{1}{2}$ of $17\frac{1}{2}$? $\frac{3}{4}$ of $33\frac{2}{3}$?
- Which is the greater, $\frac{1}{3}$ or $\frac{2}{6}$? How much greater?
- What is $\frac{3}{11}$ of 49? 50 is $\frac{3}{16}$ of what number?
- What fractions equal to $\frac{3}{8}$ have for numerators 6, 51, 33, 39, 48?

6. If $\frac{5}{16}$ of a mine is worth \$27,000, what is the entire mine worth?

7. If $3\frac{3}{4}$ yards of cloth are required to make a garment, how many such garments can be made from a piece containing 56 yards?



TRACTOR-DRAWN REAPERS

8. If a tractor-drawn reaper travels $3\frac{3}{4}$ miles per hour, how many more miles will it travel in a 12-hour day than a horse-drawn reaper that goes two miles per hour?

9. Simplify $\frac{5}{8} - \frac{1}{8} + \frac{5}{24} - \frac{3}{16} - \frac{2}{3}$.

10. A boy being asked to find the value of $8\frac{4}{5} + 2\frac{7}{9} + 3\frac{3}{5} + 4\frac{2}{3}$, gave as his answer 18. How great an error did he make?

11. Seven cars carried respectively $24\frac{1}{2}$, $18\frac{2}{3}$, $15\frac{2}{3}$, 28, $17\frac{4}{5}$, $32\frac{3}{5}$, $27\frac{2}{3}$ tons of coal. Required the number of tons carried.

12. Simplify $\frac{37}{68} \times \frac{51}{111} \times \frac{152}{187} \times \frac{153}{171}$.

13. Which is the greater, $\frac{1}{2}$ or $\frac{1}{2\frac{2}{5}}$, and how much?

14. A man owns a $\frac{7}{64}$ interest in a ranch. If he sells $\frac{1}{5}$ of his share, what fraction of the whole will he still have? If the entire ranch is worth \$300,000 what is the value of his holding?

15. A farmer had 480 bushels of corn. He put $\frac{3}{8}$ of it into one bin and the rest into another. He sold $\frac{2}{9}$ of what was in the first bin and $\frac{2}{3}$ of what was in the other. The rest he kept. How much did he keep?



MOTOR-BOAT ON THE PACIFIC

16. If a motor-boat goes 65.017 miles per hour, what is the time required to travel one mile?

17. How many minutes will it take a pipe supplying water at the rate of $2\frac{5}{8}$ gallons per minute to fill a tank holding 70 gallons?

18. Express $\frac{15}{16}$ as a fraction whose denominator is 320.

19. One man does $\frac{2}{3}$ of a piece of work and another man does $\frac{2}{7}$ of it. What fraction of the work remains to be done?

20. The regular fare from Los Angeles to San Francisco is \$15. A round-trip excursion rate is made of one single fare and a third; children at half rates. What will round-trip tickets cost for two adults and two children?

21. In 1916 Resta drove a Peugeot Special at Santa Monica, Calif., 294 miles in 3 hours, 22 minutes, 48 seconds. What was his average speed per hour?

22. A boy attends a school twelve miles from his home. He spends daily, five days in the week, one hour and a half on his way to school, and two hours on his way home from school. How much time does he spend going and coming, between September 1 and July 1? How many miles does he travel during a four-year high school course?

23. A 62-ride ticket between San Francisco and Thousand Oaks costs \$4. If a passenger rides twelve miles each trip, what does his traveling cost him per mile?

24. What will 44,000 cubic feet of gas cost at \$.90 per M?

25. At \$15 a dozen what is the cost of a single article?

26. On October 1, a gas meter reading was 32,600 cubic feet; on September 1 the reading was 30,400 cubic feet. At \$.90 per M what was the gas bill for September?

27. A pedestrian averages three miles an hour for five hours each day. How many weeks will it take him to walk one thousand miles?

28. What must be added to $\frac{3}{7}$ of $\frac{5}{8}$ in order that the sum may be equal to $\frac{9}{10}$ of $3\frac{3}{4}$?

29. When O'Donnell drove a Duesenberg Special April 8, 1916, at Corona, Calif., 301.815 miles in 3 hours, 29 minutes, 52 seconds, what was his average speed?

CHAPTER IV

DECIMALS. BILLS AND ACCOUNTS

Decimal Fractions. — A fraction whose denominator is not written, but is some power of 10, is called a *decimal fraction*, or simply a *decimal*.

Thus, a number whose value is $\frac{1}{2}$ can be written either $\frac{5}{10}$ or .5. The expression .5 is read “five tenths” and is called a *decimal*. So, also, $\frac{3}{4}$ can be written $\frac{75}{100}$ or .75. Similarly, any fraction can be expressed decimally, for, since a common fraction can be considered as an indicated division, it can be changed to the decimal form by actually dividing the numerator by the denominator. Hence we have the following rule.

Rule. — To change a common fraction to a decimal, divide the numerator by the denominator.

EXAMPLE. Express $\frac{3}{8}$ as a decimal.

$$\begin{array}{r} .375 \\ 8 \overline{)3.000} \end{array} \quad \text{that is, } \frac{3}{8} = .375.$$

Exercise

Express as decimals the following fractions :

- | | | | | |
|------------------|------------------|-------------------|--------------------|---------------------|
| 1. $\frac{1}{8}$ | 4. $\frac{3}{8}$ | 7. $\frac{7}{8}$ | 10. $\frac{3}{16}$ | 13. $\frac{15}{16}$ |
| 2. $\frac{5}{8}$ | 5. $\frac{1}{2}$ | 8. $\frac{9}{16}$ | 11. $\frac{7}{32}$ | 14. $\frac{31}{32}$ |
| 3. $\frac{1}{4}$ | 6. $\frac{3}{4}$ | 9. $\frac{1}{16}$ | 12. $\frac{5}{32}$ | 15. $\frac{63}{64}$ |

Such fractions as $\frac{1}{3}$, $\frac{4}{11}$, etc., cannot be expressed exactly as decimals but can be expressed only approximately.

EXAMPLE 1. Express $\frac{1}{3}$ as a decimal.

$$\begin{array}{r} .3333, \text{ etc.} \\ 3 \overline{)1.0000} \end{array}$$

The result, correct to three places of decimals, reads .333; correct to two decimal places, the result is .33; to one place, .3.

EXAMPLE 2. Express $\frac{2}{3}$ as a decimal.

$$\begin{array}{r} .6666, \text{ etc.} \\ 3 \overline{)2.0000} \end{array}$$

Expressed correct to three places, the answer reads .667; correct to two places, .67.

Exercise

Express as decimals correct to three places the following fractions:

1. $\frac{2}{7}$

4. $\frac{4}{9}$

7. $\frac{3}{16}$

10. $\frac{4}{15}$

13. $\frac{5}{7}$

2. $\frac{4}{7}$

5. $\frac{5}{9}$

8. $\frac{13}{32}$

11. $\frac{13}{16}$

14. $\frac{6}{7}$

3. $\frac{2}{9}$

6. $\frac{3}{11}$

9. $\frac{11}{16}$

12. $\frac{7}{9}$

15. $\frac{31}{64}$

Changing Decimals to Common Fractions

Rule. — To change a decimal to a common fraction write the decimal in fractional form and reduce to lowest terms.

EXAMPLE. Change .875 to a common fraction.

$$.875 = \frac{875}{1000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}.$$

Exercise

Write as common fractions and reduce to lowest terms:

1. .25

4. .375

7. .125

10. .9375

13. .5625

2. .50

5. .625

8. .1875

11. .3125

14. .6875

3. .75

6. .875

9. .0625

12. .4375

15. .8125

Addition and Subtraction of Decimals

Rule. — To add or subtract decimals set down the numbers so that the decimal points shall be under one another.

EXAMPLE 1. Add .578, 4.331, 14.256, .361.

$$\begin{array}{r}
 .578 \\
 4.331 \\
 14.256 \\
 .361 \\
 \hline
 19.526
 \end{array}$$

EXAMPLE 2. From 4.472 subtract 2.968.

$$\begin{array}{r}
 4.472 \\
 2.968 \\
 \hline
 1.504
 \end{array}$$

Exercise

Find the sums of the following numbers :

1. 4.372, 6.876, 9.472, 8.437, 7.437, 6.874, 4.548, 4.759
2. 5.876, 6.678, 7.436, 6.538, 5.886, 4.535, 3.488, 7.768
3. 78.89, 43.64, 79.56, 58.86, 55.64, 87.53, 34.64, 56.87
4. 567.5, 78.76, 6.876, 66.67, 555.6, 44.85, 3.675, 37.72
5. 10.86, 71.85, 6.077, 177.6, 50.08, .6558, 7.876, .8892
6. 5.608, 74.43, 7.345, 73.45, 734.5, .7345, 56.08, 560.8

From the first number in each of the following examples, subtract the second number :

- | | | |
|------------------|------------------|------------------|
| 7. 57.86, 43.67 | 13. 789.6, 465.8 | 19. 876.7, 456.8 |
| 8. 89.76, 36.69 | 14. 775.5, 564.6 | 20. 968.9, 678.8 |
| 9. 643.7, 438.5 | 15. 5.769, 3.775 | 21. 7.886, 3.532 |
| 10. 4.786, 3.462 | 16. 56.68, 33.59 | 22. 589.8, 357.9 |
| 11. 555.5, 66.6 | 17. 785.7, 69.8 | 23. 64.08, 45.09 |
| 12. 58.92, 8.98 | 18. 77.65, 46.86 | 24. 845.5, 64.8 |

Multiplication of Decimals

Rule. — To multiply one decimal by another, point off in the product a number of decimal places equal to the sum of the number of decimal places in the multiplicand and multiplier.

EXAMPLE 1. Multiply 4.3 by .6.

$$\begin{array}{r} 4.3 \\ .6 \\ \hline 2.58 \end{array}$$

EXAMPLE 2. Multiply .38 by .21.

$$\begin{array}{r} .38 \\ .21 \\ \hline 38 \\ 76 \\ \hline .0798 \end{array}$$

ORAL DRILL

- | | | | |
|--------------------|--------------------|---------------------|---------------------|
| 1. $.3 \times .5$ | 6. $16 \times .2$ | 11. $.09 \times .3$ | 16. $.8 \times 1.2$ |
| 2. $.07 \times 4$ | 7. $17 \times .2$ | 12. $.02 \times .4$ | 17. $.8 \times .12$ |
| 3. $.02 \times .5$ | 8. $.02 \times 3$ | 13. $.8 \times .9$ | 18. $.07 \times .3$ |
| 4. $.6 \times .8$ | 9. $.03 \times 7$ | 14. $.07 \times .7$ | 19. $9 \times .07$ |
| 5. $.4 \times .8$ | 10. $.5 \times .8$ | 15. $.8 \times .6$ | 20. $.6 \times .7$ |

Exercise

- | | | | |
|----------------------|-----------------------|-----------------------|-----------------------|
| 1. 67.6×5 | 6. 5.78×7.1 | 11. 65.8×7.6 | 16. 75.3×6.7 |
| 2. 83.5×8 | 7. 65.2×6.6 | 12. 688×6.5 | 17. $46.7 \times .9$ |
| 3. 5.56×4.6 | 8. 776×56.1 | 13. $538 \times .65$ | 18. $.87 \times .87$ |
| 4. 79.4×6.6 | 9. 6.5×4.7 | 14. 307×3.07 | 19. $.08 \times 1.66$ |
| 5. 835×8.34 | 10. 774×7.74 | 15. 866×5.11 | 20. $4.45 \times .06$ |

Short Method for Multiplying by 10, 100, 1000, etc.

Rules. — To multiply by 10, move the decimal point one place to the right.

To multiply by 100, move the decimal point two places to the right.

To multiply by 1000, move the decimal point three places to the right, and so on.

ORAL DRILL

Multiply each of the following numbers by 10 :

1. .36	6. 8.6	11. 6.4	16. 29	21. 2.07	26. .738
2. 6.1	7. .07	12. .75	17. 3.9	22. 21.4	27. 3.77
3. .01	8. 8.7	13. .88	18. 1.01	23. 2.33	28. 72.6
4. .65	9. 8.9	14. 7.2	19. .12	24. 32.7	29. 2.75
5. .91	10. .99	15. .67	20. 1.7	25. 41.2	30. .862

ORAL DRILL

Multiply each of the first fifteen of the following numbers by 100; multiply each of the last fifteen by 1000 :

1. .321	6. 9.86	11. 5.20	16. 6.53	21. .630	26. .581
2. 2.73	7. 2.48	12. .706	17. .742	22. 3.72	27. .047
3. .702	8. .377	13. .977	18. 2.34	23. .247	28. .030
4. .627	9. .822	14. 8.98	19. 9.05	24. .962	29. .074
5. 46.2	10. 7.64	15. .407	20. .328	25. 4.43	30. .028

Division of Decimals

Rule. — To divide one decimal by another, point off in the quotient a number of decimal places equal to the number of decimal places in the dividend minus the number of decimal places in the divisor.

EXAMPLE 1. Divide 67.51 by 4.3.

$$\begin{array}{r}
 15.7 \\
 4.3 \overline{)67.51} \\
 \underline{43} \\
 245 \\
 \underline{215} \\
 301 \\
 \underline{301} \\
 0
 \end{array}$$

It is sometimes a convenience to add one or more ciphers to the dividend to make the number of decimal places appear the same in dividend and divisor.

EXAMPLE 2. Divide 315 by 3.75.

$$\begin{array}{r}
 84 \\
 3.75 \overline{)315.00} \\
 \underline{300} \\
 15 \\
 \underline{15} \\
 0
 \end{array}$$

ORAL DRILL

Perform the following indicated divisions:

- | | | |
|-------------------|---------------------|---------------------|
| 1. $1.2 \div .3$ | 11. $.64 \div .16$ | 21. $8.1 \div 9$ |
| 2. $1.8 \div .6$ | 12. $.72 \div .9$ | 22. $100 \div 10$ |
| 3. $2.4 \div 1.2$ | 13. $7.2 \div .9$ | 23. $12.1 \div .11$ |
| 4. $.36 \div .9$ | 14. $7.2 \div 3.6$ | 24. $14.4 \div .12$ |
| 5. $.42 \div .7$ | 15. $.81 \div 9$ | 25. $16.9 \div 1.3$ |
| 6. $.48 \div .6$ | 16. $.81 \div .27$ | 26. $19.6 \div 1.4$ |
| 7. $50 \div 5$ | 17. $1.08 \div .12$ | 27. $2.25 \div 1.5$ |
| 8. $5.6 \div .8$ | 18. $1.08 \div .9$ | 28. $2.56 \div 1.6$ |
| 9. $.56 \div 7$ | 19. $12.1 \div 1.1$ | 29. $.289 \div .17$ |
| 10. $.64 \div 8$ | 20. $13.2 \div 1.2$ | 30. $.324 \div .18$ |

Exercise

Perform the following indicated divisions and obtain results correct to two decimal places :

- | | | | |
|--------------------|----------------------|----------------------|----------------------|
| 1. $345 \div 6.2$ | 6. $34.8 \div .45$ | 11. $3.25 \div .485$ | 16. $3.48 \div .311$ |
| 2. $24.3 \div 7.3$ | 7. $45.7 \div 5.2$ | 12. $8.47 \div 6.81$ | 17. $5.76 \div .256$ |
| 3. $68.2 \div .84$ | 8. $68.4 \div 6.1$ | 13. $54.5 \div 34.5$ | 18. $8.76 \div .55$ |
| 4. $71.4 \div 2.4$ | 9. $9.75 \div 7.4$ | 14. $68.5 \div 24$ | 19. $643 \div 3.14$ |
| 5. $82.7 \div 2.5$ | 10. $8.97 \div 4.37$ | 15. $7.95 \div 3.25$ | 20. $75.5 \div 5.7$ |

TABLE OF FRACTIONAL PARTS OF A DOLLAR AND THEIR EQUIVALENTS IN CENTS

FRACTION OF A DOLLAR	EQUIVALENT IN CENTS	FRACTION OF A DOLLAR	EQUIVALENT IN CENTS	FRACTION OF A DOLLAR	EQUIVALENT IN CENTS	FRACTION OF A DOLLAR	EQUIVALENT IN CENTS
$\frac{1}{100}$	1	$\frac{1}{12}$	$8\frac{1}{3}$	$\frac{1}{4}$	25	$\frac{2}{3}$	$66\frac{2}{3}$
$\frac{1}{50}$	2	$\frac{1}{10}$	10	$\frac{1}{3}$	$33\frac{1}{3}$	$\frac{3}{4}$	75
$\frac{1}{40}$	$2\frac{1}{2}$	$\frac{1}{8}$	$12\frac{1}{2}$	$\frac{3}{8}$	$37\frac{1}{2}$	$\frac{4}{5}$	80
$\frac{1}{25}$	4	$\frac{1}{6}$	$16\frac{2}{3}$	$\frac{2}{5}$	40	$\frac{5}{6}$	$83\frac{1}{3}$
$\frac{1}{20}$	5	$\frac{1}{5}$	20	$\frac{1}{2}$	50	$\frac{7}{8}$	$87\frac{1}{2}$
$\frac{1}{16}$	$6\frac{1}{4}$			$\frac{5}{8}$	$62\frac{1}{2}$		

ORAL DRILL

Perform the following indicated multiplications :

- | | | |
|----------------------------------|-----------------------------------|------------------------------------|
| 1. $\$.05 \times 20$ | 11. $\$.12\frac{1}{2} \times 24$ | 21. $\$.62\frac{1}{2} \times 40$ |
| 2. $\$.05 \times 10$ | 12. $\$.16\frac{2}{3} \times 6$ | 22. $\$.62\frac{1}{2} \times 80$ |
| 3. $\$.05 \times 200$ | 13. $\$.16\frac{2}{3} \times 18$ | 23. $\$.75 \times 12$ |
| 4. $\$.02\frac{1}{2} \times 100$ | 14. $\$.16\frac{2}{3} \times 30$ | 24. $\$.75 \times 20$ |
| 5. $\$.02\frac{1}{2} \times 10$ | 15. $\$.16\frac{2}{3} \times 60$ | 25. $\$.75 \times 50$ |
| 6. $\$.06\frac{1}{4} \times 16$ | 16. $\$.16\frac{2}{3} \times 100$ | 26. $\$.1.75 \times 40$ |
| 7. $\$.06\frac{1}{4} \times 48$ | 17. $\$.25 \times 6$ | 27. $\$.1.75 \times 50$ |
| 8. $\$.06\frac{1}{4} \times 96$ | 18. $\$.25 \times 60$ | 28. $\$.1.50 \times 72$ |
| 9. $\$.12\frac{1}{2} \times 8$ | 19. $\$.25 \times 48$ | 29. $\$.1.20 \times 60$ |
| 10. $\$.12\frac{1}{2} \times 30$ | 20. $\$.25 \times 144$ | 30. $\$.1.87\frac{1}{2} \times 24$ |

Bills and Accounts

Bill. — A detailed statement of goods bought or a statement of services rendered is called a *bill*.

EXAMPLE 1

Chicago, Ill., Sept. 7, 1919

Mr. T. B. Hatch,
1123 Vine Ave.,

*Bought of Schofield and Williams,
Hardware,
1234 Market St.*

1919				
Sept.	7	3 Pr. Roller Skates	\$2.10	\$6 30
"	"	1 Union Gas Stove	3.80	3 80
"	"	10 lb. Manila Rope	.40	4 00
"	"	1 W. J. D. Lawn Mower	7.65	7 65
"	"	1 Alpha Electric Lantern . . .	2.45	2 45
				\$24 20

Observe that a bill properly made out shows (1) the place and date of sale; (2) the names of the buyer and seller; (3) the name, quantity, and price of each article; (4) the entire amount for each item; (5) the total amount of the bill.

Exercise

Make out in proper form the following bills:

1. F. C. Bailey bought of Henry Blake & Co., Spokane, Wash., one pair of pruning shears at \$.85; two spading forks at \$1.30; 1 rake at \$.65; 1 shovel at \$1.10; 1 ax at \$1.50; 2 hoes at \$.75.

2. Mrs. H. T. McManus bought of Callahan, Grimshaw, and Fogarty, 610 Green St., New York, 16 yd. lace insertion at \$.16; $7\frac{1}{2}$ yd. pink muslin at \$.20; 1 lady's collar at \$.75; 3 spools of thread at \$.12.

3. Charles Munro bought of O. U. Armstrong, 1231 Pine St., Philadelphia, Pa., one punching bag at \$3.50; one platform for same at \$7.25; four pair of 4-lb. dumb-bells at \$1.10 per pair; one instruction book at \$.10; four pair of Indian clubs at \$.75 per pair.

4. W. J. Dolph bought of Roanoke and Smith, 1492 Columbus Ave., Denver, Colo., 1 lawn mower at \$12.25; 1 catcher for same at \$.90; 1 pair grass clippers at \$1.10; 1 can oil at \$.15; 1 screw-driver at \$.30.

5. John C. Burke bought of C. M. Adams, Stationer, 1732 Washington St., Minneapolis, 1 A. & B. Rhetoric at \$1.30; 1 Brownlee's Chemistry at \$1.50; 1 Smith's Latin Lessons at \$1.20; 2 lead pencils at \$.05; 3 R. & S. Folders at \$.60; 4 pads of paper for same at \$.10.

6. E. B. Couch bought of Herman, Sherman, and Verman, Haberdashers, 1234 Colorado St., Trenton, N. J., 1 felt hat at \$6; 2 silk shirts at \$8.50; 1 necktie at \$1.75; 2 neckties at \$.75; 2 pair silk hose at \$1.25.

7. L. A. Wisler bought of the Atlas Outfitting Co., 1024-1028 Maple Ave., Philadelphia, Pa., 1 refrigerator at \$16.50; 1 gray blanket at \$5.25; 1 oak dining table at \$19; 1 set of curtain stretchers at \$1.50; 1 oak rocker at \$11.50.

8. W. W. Keyes bought of Copperfield and Dombey, 111 Kansas Ave., Dallas, Texas, 1 parlor stand at \$8.85; 1 parlor table at \$15.50; 1 armchair at \$19.50; 1 armchair at \$14.85.

9. Mr. C. Green bought of The Keely-Davidson Supply Co., Denver, Colo., 1 gallon Walkon floor paint at \$1.89; 3 gallons Champion barn paint at \$1.15; 2 gallons A. & T. white enamel at \$3.45; 25 feet of garden hose at \$.12; 2 wash boilers at \$1.85.

Receipted Bill.— A bill is said to be *receipted* when the words “Received payment” are written at the bottom of the bill either by the seller or by some person authorized by him.

When such person signs the name of the seller he should write on the next line the word “by” or “per” and his own name or initials.

Receipted Bill

EXAMPLE 1

Detroit, Mich., Aug. 7, 1919

Mr. M. S. Moore,
123 Pine St.

*Bought of Cheeryble Bros.,
Wholesale and Retail Grocers,
412-418 Main St.*

TERMS: Cash.

1919					
Aug.	7	1 can B. & H. coffee	\$	40	
“	“	1 roll butter		50	
“	“	2 doz. eggs	\$.55	1 10	
“	“	1 watermelon		25	
“	“	1 loaf graham bread		14	
“	“	1 can Crisco		80	
				\$3	19
Received Payment, Aug. 7, 1919,					
CHEERYBLE BROS.,					
By <i>Traddles</i>					

Receipt

EXAMPLE 2

\$5.00

Ann Arbor, Mich., Sept. 14, 1919

Received from D. Atkinson

Five Dollars

For 2 hr. private instruction in
Algebra.

Richard Dana

Exercise

Make out and receipt the bills for the following transactions :

1. C. E. Locke bought of Sawyer and Finn, 42 Clemens Ave., Hannibal, Mo., 5 yards of satin at \$1.75; 6 yards of mouseline at \$2.25; 4 yards of silk at \$2; 10 yards of serge at \$1.50; 15 yards of poplin at \$2.

2. F. G. Appleton bought of The Marine Equipment Co., 108 Water St., New Orleans, La., 1 pair of army shoes at \$5.50; 1 U. S. Army olive-drab sweater at \$5; 1 U. S. razor at \$1.50; 2 pairs canvas lace leggings at \$.95.

3. M. J. Mulvaney bought of Levinsky and O'Dowd, Jewelers, Mexico City, Mo., 1 24-jewel watch at \$65; 1 watch fob, gold panel, at \$12.50; 1 pair pearl earrings at \$26.65; 1 signet ring at \$8.50; 1 tourmaline stick pin at \$14.

4. Mr. Geo. McDaniels bought of C. Stratford, Attleboro, Vt., 1 velvet rug 9×12 at \$67.95; 1 Brussels rug 6×9 at \$28.75; 1 tapestry rug 8.3×10.6 at \$26.75; 2 tapestry Brussels rugs 6×9 at \$21.45.

5. Mr. C. H. Thorpe bought of Bibb, Newman, and Ikenberg, 66 Central Ave., Boston, Mass., 1 library table at \$8.80; 1 oak extension table at \$12.75; 1 leather rocker at \$17.75.

6. A. L. Tunison bought of The U. S. Grocery Co., 422 Tehama St., San Francisco, Calif., 2 packages of flapjack flour at \$.14; 1 package Pillsbury's Vitos at \$.15; 2 packages of Spee-Dee cleaner at \$.20; 1 large can Crisco at \$1.40; 5 pounds Hotel Blend coffee at \$.19; 1 package Instant Postum at \$.40; 3 packages of Puffed Wheat at \$.13; 1 pound Ridge-way's 5 o'clock tea at \$.64; 4 pounds of Ghirardelli's Select Eagle chocolate at \$.30.

7. Receipt the bills made out in the Exercise on pages 61 and 62.

Debt and Credit. — The amount which one person owes another is called a *debt*.

The amount due a person or the sum paid toward discharging a debt is called a *credit*.

The person who owes is called the *debtor*. The person to whom an amount is owed is called a *creditor*.

Account. — An *account* implies a business transaction covering a period of time in which there are both a debtor's *bill* and a debtor's *payments*.

Statement. — It is customary for the creditor to send an itemized account to the debtor. If it is not paid at that time, another form of bill, called a *statement*, is sent. This contains only the words: "To account rendered" or "To Mdse." followed by the amount. Statements are usually sent out on the first of the month.

Form of Account

EXAMPLE 1

Detroit, Mich., Oct. 1, 1919

Mr. H. L. Detrick

*In account with
Kelly, Fitzpatrick & Connelly.*

Sept.	1	To account rendered . . .	\$ 124 30		
"	12	5 doz. plates, 8 in., at . . \$ 1.50	7 50		
"	24	35 doz. plates, 7 in., at . . 1.35	47 25		
"	27	10 doz. plates, 5 in., at . . .90	9 00	\$ 188 05	
"	14	By cash	100 00		
"	20	By cash	45 00	145 00	
				\$ 43 05	

If the above balance were paid in full Oct. 1, some such words as the following would be written at the bottom :

"Received payment, Oct. 1, 1919,
Kelly, Fitzpatrick & Connelly,
By T. Bone."

Observe that the account sets forth the place and date of sale, names of buyer and seller, etc., as in the case of the original bill.

Statement

EXAMPLE 2

St. Louis, Mo., Aug. 1, 1919

Mr. W. H. Wagner,
432 Second St.

*In account with Olsen and Larsen,
Grocers.*

July	1	Account rendered	\$ 14	40		
"	3	White King soap		25		
"	"	6 pkg. matches		25		
"	"	4 cans T. soup at \$.11		44		
"	10	2 lb. spaghetti		20		
"	"	3 pkg. Q. Oats 10		30		
"	17	100 lb. beet sugar	7	95		
"	"	100 lb. cane sugar	8	10	\$ 31	89
		CR.				
"	14	4 cans T. soup		44		
"	"	Cash	25		25	44
					\$ 6	45

Exercise

Render the following accounts :

1. Mrs. Gustav Petersen bought of Mulvaney and Ortheris, of 1918 Union St., St. Paul, Minn., on

June 17 4 yd. of 34-inch flouncing at \$1.00

June 19 4 yd. of 18-inch flouncing at .75

June 21 1 gingham house dress 2.45

Paid June 25 \$5 on account.

2. Ben Gunn of Treasure Island bought of Stevenson Creamery, 44 Ocean Ave., H. I.,

Apr. 1	2 lb. Roquefort cheese	at \$ 1.25
" "	2½ lb. Swiss cheese	at .46
" "	3 lb. Tillamook cheese	at .30
" "	4 lb. Wisconsin cheese	at .34

Paid Apr. 1 on account \$ 3.30.

3. E. H. Skinner bought of David Eklund & Co., Baltimore, Md.,

Oct. 3	1 rocking chair	at \$ 7.65
" 14	2 wall mirrors	at 12.50
" 19	2 Morris chairs	at 62.50

Paid \$ 20 on Oct. 5 ; \$ 10 on Oct. 15 ; \$ 20 on Oct. 19 ; \$ 25 on Oct. 25.

4. Wesley Iberg bought of the Polaris Electric Co., St. Paul, Minn.,

Sept. 19	1 set of lead plates	at \$ 6.00
" "	Recharging of battery	.75
" "	4 days' rent of battery	at \$.25 per day

Paid in full Sept. 19.

5. Fred Frover purchased of the Homer Brown Supply Co., 4272 State St., Akron, Ohio,

Dec. 24	1 safety razor	\$ 5.00
" "	1 box talcum powder	.25
" "	1 hand mirror	1.75
" "	12 oz. of witch hazel	.25

Paid on account Dec. 24, \$ 5.

CHAPTER V

PERCENTAGE

Per cent means *by the hundred*. Hence, 5 per cent of a number means at the rate of 5 out of a hundred; 25 per cent means 25 out of a hundred, and so on.

Per cent is usually indicated by the symbol $\%$. Thus, 8% means 8 per cent, or 8 out of a hundred, or 8 hundredths. It can be written also $\frac{8}{100}$ or .08. Seven and one half per cent can be written $7\frac{1}{2}\%$ or .075. Thus, per cent can be expressed either as a decimal fraction or as a common fraction.

Rule. — To express per cent as a common fraction, (1) write the number indicating the per cent for the numerator and 100 for the denominator, and (2) reduce this fraction to lowest terms.

EXAMPLE. Express 5% both as a decimal and as a common fraction in its lowest terms.

$$5\% = .05 = \frac{5}{100} = \frac{1}{20}.$$

ORAL DRILL

Express as decimals and as common fractions in their lowest terms the following per cents:

- | | | | | |
|---------|----------|-----------------------|-----------------------|-----------------------|
| 1. 10 % | 5. 75 % | 9. 200 % | 13. $12\frac{1}{2}\%$ | 17. $87\frac{1}{2}\%$ |
| 2. 20 % | 6. 100 % | 10. 275 % | 14. $8\frac{1}{3}\%$ | 18. $\frac{1}{2}\%$ |
| 3. 25 % | 7. 120 % | 11. $33\frac{1}{3}\%$ | 15. $16\frac{2}{3}\%$ | 19. $\frac{1}{10}\%$ |
| 4. 50 % | 8. 150 % | 12. $66\frac{2}{3}\%$ | 16. $62\frac{1}{2}\%$ | 20. $\frac{3}{4}\%$ |

Exercise

Write the following decimals as fractions and as per cents :

- | | | | | |
|--------|----------------------|----------------------|-----------------------|-----------------------|
| 1. .05 | 5. .75 | 9. .16 $\frac{2}{3}$ | 13. 3.00 | 17. .06 $\frac{2}{3}$ |
| 2. .10 | 6. .33 $\frac{1}{3}$ | 10. .375 | 14. 9.00 | 18. .03 $\frac{1}{3}$ |
| 3. .20 | 7. .66 $\frac{2}{3}$ | 11. 1.50 | 15. 1.10 | 19. .005 |
| 4. .50 | 8. .12 $\frac{1}{2}$ | 12. 2.75 | 16. .08 $\frac{1}{3}$ | 20. .001 |

Exercise

Write the following fractions as decimals and as per cents :

- | | | | | |
|------------------|------------------|--------------------|--------------------|--------------------|
| 1. $\frac{1}{2}$ | 5. $\frac{1}{6}$ | 9. $\frac{5}{8}$ | 13. $\frac{1}{20}$ | 17. $\frac{5}{6}$ |
| 2. $\frac{1}{3}$ | 6. $\frac{2}{3}$ | 10. $\frac{7}{8}$ | 14. $\frac{1}{24}$ | 18. $\frac{1}{9}$ |
| 3. $\frac{1}{4}$ | 7. $\frac{3}{4}$ | 11. $\frac{1}{8}$ | 15. $\frac{1}{25}$ | 19. $\frac{1}{11}$ |
| 4. $\frac{1}{5}$ | 8. $\frac{3}{8}$ | 12. $\frac{1}{12}$ | 16. $\frac{1}{16}$ | 20. $\frac{1}{30}$ |

The following is a tabulated list of the percentage equivalents of commonly occurring fractions :

$\frac{1}{100} = 1\%$	$\frac{1}{15} = 6\frac{2}{3}\%$	$\frac{1}{7} = 14\frac{2}{7}\%$	$\frac{1}{2} = 50\%$
$\frac{1}{50} = 2\%$	$\frac{1}{12} = 8\frac{1}{3}\%$	$\frac{1}{6} = 16\frac{2}{3}\%$	$\frac{5}{8} = 62\frac{1}{2}\%$
$\frac{1}{30} = 3\frac{1}{3}\%$	$\frac{1}{11} = 9\frac{1}{11}\%$	$\frac{1}{5} = 20\%$	$\frac{2}{3} = 66\frac{2}{3}\%$
$\frac{1}{25} = 4\%$	$\frac{1}{10} = 10\%$	$\frac{1}{4} = 25\%$	$\frac{3}{4} = 75\%$
$\frac{1}{20} = 5\%$	$\frac{1}{9} = 11\frac{1}{9}\%$	$\frac{1}{3} = 33\frac{1}{3}\%$	$\frac{5}{6} = 83\frac{1}{3}\%$
$\frac{1}{16} = 6\frac{1}{4}\%$	$\frac{1}{8} = 12\frac{1}{2}\%$	$\frac{3}{8} = 37\frac{1}{2}\%$	$\frac{7}{8} = 87\frac{1}{2}\%$

ORAL DRILL

1. What is 50% of 100? of 80? of 60? of 360? of 500? of 1000?

2. What is 25% of 100? of 80? of 60? of 360? of 500? of 1000?

3. What is 10% of 100? of 80? of 60? of 360? of 500? of 1000?

4. What is 20% of 100? of 80? of 60? of 360? of 500? of 1000?

5. What is 5 % of 100? of 80? of 60? of 360? of 500? of 1000?

Base, Rate, Percentage

The **base** is the number on which the percentage is computed.

The **rate** is the number of hundredths and is usually expressed as a decimal.

The **percentage** is the result obtained by taking a certain per cent of the base.

Rule. — To find the percentage when the base and rate are given, express the rate as a decimal and multiply the base by it.

EXAMPLE 1. Find 6 % of \$50.

\$50 the Base, is the number on which the percentage is computed.

.06 the Rate, is the number of hundredths taken.

\$3.00 the Percentage, is the result of multiplying base by rate.

EXAMPLE 2. Find 52 % of \$420.

$$\begin{array}{r}
 \$420 \text{ Base} \\
 \underline{.52 \text{ Rate}} \\
 840 \\
 2100 \\
 \hline
 \$218.40 \text{ Percentage}
 \end{array}$$

EXAMPLE 3. Find $31\frac{1}{2}$ % of \$150.

$$\begin{array}{r}
 .315 \text{ Rate} \\
 \underline{\$150 \text{ Base}} \\
 15750 \\
 315 \\
 \hline
 \$472.50 \text{ Percentage}
 \end{array}$$

ORAL DRILL

- | | |
|---------------------|----------------------|
| 1. Find 3 % of 100 | 7. Find 18 % of 20 |
| 2. Find 4 % of 200 | 8. Find 22 % of 30 |
| 3. Find 6 % of 1000 | 9. Find 26 % of 300 |
| 4. Find 10 % of 500 | 10. Find 54 % of 200 |
| 5. Find 12 % of 600 | 11. Find 41 % of 20 |
| 6. Find 14 % of 20 | 12. Find 61 % of 30 |

- | | |
|-----------------------|----------------------|
| 13. Find 82 % of 40 | 22. Find 62 % of 50 |
| 14. Find 95 % of 1000 | 23. Find 74 % of 50 |
| 15. Find 9 % of 200 | 24. Find 81 % of 600 |
| 16. Find 19 % of 200 | 25. Find 65 % of 20 |
| 17. Find 29 % of 200 | 26. Find 46 % of 200 |
| 18. Find 8 % of 60 | 27. Find 57 % of 40 |
| 19. Find 18 % of 30 | 28. Find 82 % of 30 |
| 20. Find 28 % of 20 | 29. Find 85 % of 40 |
| 21. Find 52 % of 40 | 30. Find 96 % of 200 |

Exercise

1. What is $6\frac{1}{4}$ % of 100 ? of 96 ? of 360 ? of 500 ? of 600 ? of 1 ?
2. What is $6\frac{2}{3}$ % of 100 ? of 96 ? of 360 ? of 500 ? of 600 ? of 1 ?
3. What is $3\frac{1}{3}$ % of 100 ? of 96 ? of 360 ? of 500 ? of 600 ? of 1 ?
4. What is $14\frac{2}{7}$ % of 100 ? of 96 ? of 360 ? of 500 ? of 600 ? of 1 ?
5. What is $83\frac{1}{3}$ % of 100 ? of 96 ? of 360 ? of 500 ? of 600 ? of 1 ?
6. What is $87\frac{1}{2}$ % of 100 ? of 96 ? of 360 ? of 500 ? of 600 ? of 1 ?
7. What is $37\frac{1}{2}$ % of 100 ? of 96 ? of 360 ? of 500 ? of 600 ? of 1 ?
8. What is $62\frac{1}{2}$ % of 100 ? of 96 ? of 360 ? of 500 ? of 600 ? of 1 ?
9. What is $66\frac{2}{3}$ % of 100 ? of 96 ? of 360 ? of 500 ? of 600 ? of 1 ?
10. What is $87\frac{1}{2}$ % of 100 ? of 96 ? of 360 ? of 500 ? of 600 ? of 1 ?

Exercise

- | | |
|---------------------|----------------------|
| 1. Find 12 % of 46 | 9. Find 64 % of 212 |
| 2. Find 18 % of 72 | 10. Find 68 % of 654 |
| 3. Find 22 % of 112 | 11. Find 72 % of 56 |
| 4. Find 28 % of 256 | 12. Find 76 % of 62 |
| 5. Find 35 % of 448 | 13. Find 84 % of 212 |
| 6. Find 41 % of 64 | 14. Find 88 % of 256 |
| 7. Find 44 % of 69 | 15. Find 95 % of 864 |
| 8. Find 55 % of 115 | 16. Find 97 % of 978 |

Rule. — To find the rate when the base and percentage are given, divide the percentage by the base.

EXAMPLE 1. What per cent of \$400 is \$28?

	.07	Rate
Base 400	28.00	Percentage
	28 00	

EXAMPLE 2. What per cent of \$526 is \$39.45?

	.075	Rate
Base 526	39.450	Percentage
	36 82	
	2 630	
	2 630	

ORAL DRILL

- | | |
|--------------------------------|---|
| 1. 30 is what per cent of 60? | 7. 70 is what per cent of 200? |
| 2. 33 is what per cent of 66? | 8. 50 is what per cent of 60? |
| 3. 10 is what per cent of 40? | 9. 22 is what per cent of 88? |
| 4. 40 is what per cent of 10? | 10. $22\frac{1}{2}$ is what per cent of 90? |
| 5. 15 is what per cent of 30? | 11. 60 is what per cent of 50? |
| 6. 60 is what per cent of 200? | 12. 75 is what per cent of 25? |

13. 25 is what per cent of 75? 17. 30 is what per cent of 300?
 14. 20 is what per cent of 50? 18. 45 is what per cent of 250?
 15. 50 is what per cent of 20? 19. 55 is what per cent of 220?
 16. 20 is what per cent of 60? 20. $11\frac{1}{4}$ is what per cent of 45?

Exercise

1. 32 is what per cent of 42? 11. 65 is what per cent of 91?
 2. 34 is what per cent of 40? 12. 68 is what per cent of 112?
 3. 45 is what per cent of 62? 13. 72 is what per cent of 200?
 4. 52 is what per cent of 76? 14. 72 is what per cent of 39?
 5. 55 is what per cent of 205? 15. 72 is what per cent of 170?
 6. 58 is what per cent of 192? 16. 41 is what per cent of 81?
 7. 64 is what per cent of 96? 17. 81 is what per cent of 41?
 8. 64 is what per cent of 48? 18. 92 is what per cent of 93?
 9. 64 is what per cent of 65? 19. 94 is what per cent of 36?
 10. $37\frac{1}{2}$ is what per cent of 45? 20. 98 is what per cent of 110?

Rule. — To find the base when the percentage and the rate are given, divide the percentage by the rate.

EXAMPLE 1. 378 is 18% of what number?

$$\begin{array}{r} \text{Rate } .18 \quad \begin{array}{|l} 2100 \text{ Base} \\ \hline 378.00 \text{ Percentage} \\ 36 \\ \hline 1800 \end{array} \end{array}$$

EXAMPLE 2. \$42 is $3\frac{1}{2}\%$ of what?

$$\begin{array}{r} \text{Rate } .035 \quad \begin{array}{|l} 1200 \text{ Base} \\ \hline 42.000 \text{ Percentage} \\ 35 \\ \hline 7000 \end{array} \end{array}$$

ORAL DRILL

1. 14 is 50% of what number?
2. 30 is 25% of what number?
3. 30 is 75% of what number?
4. 36 is 10% of what number?
5. 20 is 5% of what number?

6. 600 is 60 % of what number?
7. 12 is $12\frac{1}{2}$ % of what number?
8. 22 is $16\frac{2}{3}$ % of what number?
9. 11 is $8\frac{1}{3}$ % of what number?
10. 10 is $6\frac{1}{4}$ % of what number?
11. 30 is $37\frac{1}{2}$ % of what number?
12. 50 is $62\frac{1}{2}$ % of what number?
13. 50 is $83\frac{1}{2}$ % of what number?
14. 700 is $87\frac{1}{2}$ % of what number?
15. 3.5 is $12\frac{1}{2}$ % of what number?
16. 35 is $12\frac{1}{2}$ % of what number?
17. .5 is $62\frac{1}{2}$ % of what number?
18. \$.30 is $16\frac{2}{3}$ % of what number?
19. \$2.50 is $33\frac{1}{3}$ % of what number?
20. \$10.50 is 25 % of what number?

Exercise

1. 12 is 15 % of what number?
2. 14 is 16 % of what number?
3. 22 is 24 % of what number?
4. 26 is 8 % of what number?
5. 36 is 16 % of what number?
6. 42 is 12 % of what number?
7. 45 is 18 % of what number?
8. 52 is 22 % of what number?
9. 54 is 24 % of what number?
10. 56 is 26 % of what number?
11. 58 is 42 % of what number?
12. 62 is 55 % of what number?

13. 64 is $14\frac{1}{2}\%$ of what number?
14. 68 is 82% of what number?
15. 72 is 87% of what number?
16. 61 is $22\frac{1}{2}\%$ of what number?
17. 75 is $32\frac{1}{2}\%$ of what number?
18. 82 is 91% of what number?
19. 91 is 95.5% of what number?
20. 94 is 110% of what number?

Profit and Loss

Net and Gross Cost. — The amount actually paid for an article is called its *prime* or *net cost*.

When transportation charges, insurance, commission for buying, etc., are added, the result is the *gross cost*.

Net and Gross Profit. — The difference between the gross cost and the actual selling price is the *gross profit*. The gross profit less the selling costs such as salaries, traveling expenses, commissions, etc., is the *net profit*. In case the gross cost is greater than the net selling price, there is a loss.

It is pretty generally the custom among merchants that the cost of operating is figured upon the sales. Both the net and gross gains are figured upon the sales.

EXAMPLE. If a piece of property is bought for \$3500 and sold for \$4200, what is the gain per cent?

\$ 4200	Selling price		
3500	Cost	Cost 3500	.20 700.00 700 0
\$ 700	Profit		Profit
			0

20 % gain

Exercise

1. From a farm of 160 acres, 30 acres were sold. What per cent was sold?
2. If eggs are bought at 60 cents a dozen and sold at 75 cents a dozen, what is the per cent profit?
3. A real estate dealer bought three lots for \$2000, \$2800, and \$3200 respectively. He sold the first at an advance of 8 %, the second at an advance of 10 %, and the third at an advance of 15 %. What was his total gain?
4. A dealer bought 600 bushels of wheat at \$1.84 and 800 bushels at \$1.86. He sold it all at \$2.05 a bushel. What was his gain per cent?
5. If a rug is bought for \$50 and sold for \$112.50, what is the percentage of profit?
6. The excavation of a certain cellar cost \$175. At the same rate what is the cost of an excavation 20 % wider, 50 % longer, and 25 % deeper?
7. If chairs are bought at \$25 per dozen and sold at \$2.75 apiece, what is the percentage of profit?
8. If men's hats are bought for \$28 a dozen and retailed at \$3.25 each, what per cent gain is made?
9. If pocket knives are bought at \$15 a dozen and sold at \$1.75 apiece, what is the percentage of gain?
10. If lead pencils are bought for \$3.50 a gross and sold at 5 cents apiece, what is the per cent gain?
11. If notebooks are bought at \$9 a gross and sold at 10 cents each, what percentage of profit is made?
12. If cartridges are bought for \$12 a thousand and sold in boxes of 50 each at \$1.75 per box, what is the per cent profit?

13. If a piece of property is bought for \$6000 and the taxes and insurance on it are \$96 annually, what must the monthly rental be in order that the return may be 12 % of the investment?

14. Find the corresponding selling prices if goods are marked as follows:

16 $\frac{2}{3}$ % off on goods marked \$ 2.50

33 $\frac{1}{3}$ % off on goods marked \$ 4.75

66 $\frac{2}{3}$ % off on goods marked \$ 18.50

12 $\frac{1}{2}$ % off on goods marked \$ 26.00

37 $\frac{1}{2}$ % off on goods marked \$ 16.85

15. Which is the greater: 42 % of 51 or 51 % of 42? The difference is how many per cent less than 42? The difference is how many per cent less than 51?

16. In a school whose enrollment is 525, 18 pupils were absent on Monday, 17 on Tuesday, 14 on Wednesday, 19 on Thursday, and 22 on Friday. What was the percentage of attendance for each day? What was the average percentage of attendance for the week?

17. In room *A* are 33 pupils, 31 of whom have been present every day; in room *B* are 42 pupils, 39 of whom have been present every day; in room *C* are 26 pupils, 24 of whom have been present every day. What room has the highest percentage of attendance?

18. An orchardist planted 120 trees of which 14 died; he planted 70 raspberry bushes and lost none; he planted 1000 grapevines and lost 35. What per cent of each kind planted survived? What per cent of the entire number of trees and plants set out lived?

19. A nurseryman bought 1000 avocado seeds at ten cents apiece. He figured that 10 % of them would not germinate and that frost would kill 15 % of the young trees. For how much should he sell each tree to clear \$ 1000?

20. If 58 members of a class are boys and 67 are girls, what is the percentage of each?

21. If 217 words out of 250 are spelled correctly, what is the percentage of correctly spelled words?



STEEL WORKS AT PUEBLO, COLORADO, WHERE IRON ORE IS CONVERTED INTO STEEL

22. If iron ore yields 62 % of pure metal to the ton, how much iron can be obtained from 40 tons of ore?

23. Two men form a partnership. One agrees to furnish 40 % of the capital and the other 60 %. If the first furnishes \$2500, what does the other furnish?

24. A farmer received 40 % of a crop as rent for his land. If his share of the crop amounted to 4000 bushels, what was the amount of the entire crop?

25. A man owning a $\frac{2}{3}$ interest in a mine sold $\frac{1}{2}$ of his interest. What per cent of the mine did he still own?

26. If wages are increased 15 %, what is the wage of a man who has been getting \$ 62 a week ?

27. What is the percentage of error in taking one yard equal to 35 inches ?

28. In a solder made up of 5 parts zinc, 2 parts tin, and 1 part lead, what is the percentage of each constituent ?

29. The following amounts of metal are melted together to form a soldering alloy : 18 pounds of tin, $37\frac{1}{2}$ pounds of lead, 75 pounds of bismuth, and $19\frac{1}{2}$ pounds of cadmium. What is the percentage of each ingredient ?

30. A 100-gallon mixture of alcohol and water is 70 % alcohol. How much alcohol must be added that the percentage of alcohol be increased to 80 % ?

Discount

Commercial discount is a reduction from the list price of an article or from the amount of a bill of goods.

The net price is the amount of the bill after the discount has been made.

EXAMPLE. What is the net amount of a bill of \$ 325 after a discount of 20 % has been deducted ?

Amount of bill	\$ 325
Less 20 % or $\frac{1}{5}$ of \$ 325 . . .	65
Net amount	<u>\$ 260</u>

Exercise

Find the net prices of goods billed as follows, with discounts as stated :

- | | |
|-----------------------------|--------------------------------|
| 1. \$ 25 10 % off | 6. \$ 62.50 20 % off |
| 2. \$ 14 20 % off | 7. \$ 49 5 % off |
| 3. \$ 62 15 % off | 8. \$ 55.50 2 % off |
| 4. \$ 70 10 % off | 9. \$ 150 8 % off |
| 5. \$ 48 15 % off | 10. \$ 206 6 % off |

11. \$42.50 . . . 10 % off	21. \$95.25 . . . 10 % off
12. \$32.25 . . . 20 % off	22. \$48.75 . . . 12½ % off
13. \$75.60 . . . 25 % off	23. \$115 . . . 16⅔ % off
14. \$85.10 . . . 5 % off	24. \$242.50 . . . 20 % off
15. \$412.30 . . . 8 % off	25. \$260.50 . . . 5 % off
16. \$37.85 . . . 12½ % off	26. \$512.75 . . . 12 % off
17. \$112 . . . 2 % off	27. \$675.25 . . . 2 % off
18. \$876 . . . 5 % off	28. \$325.25 . . . 3 % off
19. \$950.50 . . . 6 % off	29. \$175.75 . . . 4 % off
20. \$1250 . . . 2 % off	30. \$255.75 . . . 2 % off

Interest

Interest is money paid for the use of money.

The principal is the sum on which the interest is computed.

The amount is the principal plus the interest.

The rate is the number of hundredths of the principal paid for its use for one year.

Simple Interest

Interest is ordinarily computed on the basis of 12 months of 30 days each, or 360 days. It is then called **simple interest** or **common interest**.

Rule.—To find the interest multiply together the principal, the rate, and the time expressed in years.

EXAMPLE 1. Find the interest on \$575 for 3 years at 4 %.

$$\begin{array}{r}
 \$575 \text{ Principal} \\
 .04 \text{ Rate} \\
 \hline
 \$23.00 \text{ Interest for 1 yr.} \\
 3 \text{ Time (in years)} \\
 \hline
 \$69.00 \text{ Interest for 3 yr.}
 \end{array}$$

EXAMPLE 2. Find the interest on \$ 288 for 2 yr. 7 mo. at 5 %.

$$\begin{array}{r}
 \$ 288 \text{ Principal} \\
 \underline{.05 \text{ Rate}} \\
 \$ 14.40 \text{ Interest for 1 yr.} \\
 \underline{2 \text{ Time (in years)}} \\
 \$ 28.80 \text{ Interest for 2 yr.} \\
 7.20 \text{ Interest for 6 mo.} \\
 \underline{1.20 \text{ Interest for 1 mo.}} \\
 \$ 37.20 \text{ Interest for 2 yr. 7 mo.}
 \end{array}$$

EXAMPLE 3. Find the amount if \$ 475 is at interest for 4 yr. at 6 %.

$$\begin{array}{r}
 \$ 475 \text{ Principal} \\
 \underline{.06 \text{ Rate}} \\
 \$ 28.50 \text{ Interest for 1 yr.} \\
 \underline{4 \text{ Time (in years)}} \\
 \$ 114.00 \text{ Interest for 4 yr.} \\
 \$ 475.00 \text{ Principal} \\
 \hline
 \$ 589.00 \text{ Amount}
 \end{array}$$

Exercise

Find the interest and amount in each of the following :

1. \$ 100 for 1 year, 6 months, at 5 %
2. \$ 125 for 6 years, 6 months, at 7 %
3. \$ 150 for 5 years, 9 months, at 7 %
4. \$ 50 for 4 years, 6 months, at 6 %
5. \$ 1000 for 7 months, 15 days, at 7 %
6. \$ 350 for 3 years, 9 months, at 7 %
7. \$ 1500 for 2 months, at 6 %
8. \$ 725 for 1 year, 2 months, at 6 %
9. \$ 1200 for 2 years, 5 months, at 5 %
10. \$ 500 for 2 years, 3 months, 15 days, at 4 %

Short Methods

Frequently interest at 6 % can be computed much more easily by what is known as the *Bankers' Sixty-Day Method*, the rule for which is given below.



PARIS STOCK EXCHANGE

Rule. — To find the interest for 60 days move the decimal point in the principal two places to the left; for 6 days move the decimal point in the principal three places to the left. From these results find the interest for the required time.

EXAMPLE 1. Find the interest on \$ 840 for 4 months, 24 days, at 6 %.

\$ 8.40 Interest for 60 days (or 2 months)	
<u>2</u>	\$.840 Interest for 6 days
\$ 16.80 Interest for 4 months	<u>4</u>
3.36 Interest for 24 days	\$ 3.36 Interest for 24 days
<u>\$ 20.16</u> Interest for 4 months, 24 days	

EXAMPLE 2. Find the interest on \$ 420 for 8 months, 15 days, at 6 %.

\$ 4.20 Interest for 60 days (or 2 months)	
<u>4</u>	\$.420 Interest for 6 days
\$ 16.80 Interest for 8 months	<u>2½</u>
1.05 Interest for 15 days	\$ 1.050 Interest for 15 days
\$ 17.85 Interest for 8 months, 15 days	

Exercise

In each of the following examples find the interest at 6 % for the time indicated :

- | | |
|-----------------------|-----------------------------------|
| 1. \$ 620 for 60 days | 10. \$ 500 for 2 months, 15 days |
| 2. \$ 175 for 30 days | 11. \$ 1000 for 3 months, 12 days |
| 3. \$ 300 for 90 days | 12. \$ 850 for 4 months, 12 days |
| 4. \$ 200 for 90 days | 13. \$ 650 for 5 months, 18 days |
| 5. \$ 600 for 80 days | 14. \$ 880 for 6 months, 24 days |
| 6. \$ 750 for 90 days | 15. \$ 720 for 3 months, 12 days |
| 7. \$ 650 for 30 days | 16. \$ 850 for 6 months, 16 days |
| 8. \$ 700 for 60 days | 17. \$ 900 for 8 months, 20 days |
| 9. \$ 820 for 90 days | 18. \$ 975 for 9 months, 24 days |

When interest is computed at 4%, 5%, 7 %, or 8 % we may use the following rules.

Rules. — To find interest at 4 % decrease the interest at 6 % by $\frac{1}{3}$ of itself.

To find interest at 5 % decrease the interest at 6 % by $\frac{1}{6}$ of itself.

To find interest at 7 % increase the interest at 6 % by $\frac{1}{6}$ of itself.

To find interest at 8 % increase the interest at 6 % by $\frac{1}{3}$ of itself.

Exercise

Compute the interest for each of the following:

	PRINCIPAL	TIME	RATE
1.	\$ 80.00	6 mo. 15 da.	4 %
2.	\$ 850.00	11 mo. 15 da.	4 %
3.	\$ 2000.00	10 mo. 10 da.	5 %
4.	\$ 96.50	4 yr. 6 mo.	5 %
5.	\$ 625.75	2 yr. 4 mo. 15 da.	5 %
6.	\$ 500.00	2 yr. 8 mo. 10 da.	7 %
7.	\$ 440.00	1 yr. 2 mo. 10 da.	7 %
8.	\$ 4250.00	1 yr. 5 mo. 15 da.	8 %
9.	\$ 1500.00	3 yr. 6 mo. 15 da.	8 %
10.	\$ 75.00	1 yr. 9 mo. 20 da.	8 %

When interest is computed at $4\frac{1}{2}$ %, $7\frac{1}{2}$ %, or other fractional rate we may use the following rules.

Rules. — To find interest at $4\frac{1}{2}$ % decrease the interest at 6 % by $\frac{1}{4}$ of itself. To find interest at $7\frac{1}{2}$ % increase the interest at 6 % by $\frac{1}{4}$ of itself.

Exercise

In the first five of the following examples find the interest at $4\frac{1}{2}$ %, in the last five, at $7\frac{1}{2}$ % :

- | | |
|-----------------------|----------------------------------|
| 1. \$ 650 for 60 days | 6. \$ 550 for 2 months, 15 days |
| 2. \$ 175 for 30 days | 7. \$ 1550 for 3 months, 12 days |
| 3. \$ 475 for 90 days | 8. \$ 850 for 4 months, 12 days |
| 4. \$ 250 for 90 days | 9. \$ 1250 for 5 months, 18 days |
| 5. \$ 825 for 90 days | 10. \$ 725 for 6 months, 15 days |

Persons who have much computing of interest to do use interest tables of various kinds. In determining the amount of time from one date to another, for example, the following table is used.

BANKERS' TIME TABLE

FROM ANY DAY OF	TO THE SAME DAY OF THE NEXT											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
Mar.	306	337	365	31	61	92	122	153	184	214	245	275
Apr.	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

Compound Interest

Compound Interest is interest computed for stated intervals on the sum of the principal and unpaid interest. Customarily interest is compounded either annually or semi-annually.

EXAMPLE. To what will \$ 400 amount in 2 years, 6 months, at 4 %, compounded annually ?

Principal	\$ 400.00	
Interest for 1st yr. at 4 %	16.00	\$ 416
Principal for 2d yr.	\$ 416.00	.04
Interest for 2d yr. at 4 %	16.64	\$ 16.64
Principal for 3d yr.	\$ 432.64	
Interest for 6 mo. at 4 %	8.65	
Amount for 2 yr., 6 mo.	\$ 441.29	

Exercise

Find the amount at compound interest for each of the following:

1. \$ 100 for 2 yr. at 4 %
2. \$ 200 for 3 yr. 6 mo. at 6 %
3. \$ 300 for 2 yr. 6 mo. at 5 %
4. \$ 400 for 2 yr. 6 mo. at 7 %
5. \$ 2500 for 2 yr. at 5 %
6. \$ 3500 for 3 yr. at 6 %
7. \$ 4200 for $2\frac{1}{2}$ yr. at 4 %
8. \$ 75 for $3\frac{1}{2}$ yr. at 7 %

Compound interest is ordinarily figured from a table such as the one given below.

COMPOUND INTEREST TABLE

Amount of \$ 1, at Various Rates, Interest Compounded Annually.

YEARS	4 %	5 %	6 %	7 %
1	1.040000	1.050000	1.060000	1.070000
2	1.081600	1.102500	1.123600	1.144900
3	1.124864	1.157625	1.191016	1.225043
4	1.169859	1.215506	1.262477	1.310796
5	1.216653	1.276282	1.338226	1.402552
6	1.265319	1.340096	1.418519	1.500730
7	1.315932	1.407100	1.503630	1.605782
8	1.368569	1.477455	1.593848	1.718186
9	1.423312	1.551328	1.689479	1.838459
10	1.480244	1.628895	1.790848	1.967151
11	1.539454	1.710339	1.898299	2.104852
12	1.601032	1.795856	2.012197	2.252192
13	1.665074	1.885649	2.132928	2.409845
14	1.731676	1.979932	2.260904	2.578534
15	1.800944	2.078928	2.396558	2.759032
16	1.872981	2.182875	2.540352	2.952164
17	1.947901	2.292018	2.692773	3.158815
18	2.025817	2.406619	2.854339	3.379932
19	2.106849	2.526950	3.025600	3.616528
20	2.191123	2.653298	3.207136	3.869684

EXAMPLE. Find the sum to which \$ 700 will amount in 15 years if put at interest at 4 %, compounded annually.

\$ 1.800944 Amount of \$ 1 at 4 % for 15 years

700

\$ 1260.66 Amount of \$ 700 at 4 % for 15 years

Exercise

Using the table on page 87, find the amount in each of the following examples if the interest is compounded annually :

- | | |
|-----------------------------|------------------------------|
| 1. \$ 200 for 4 yr. at 4 % | 6. \$ 100 for 8 yr. at 5 % |
| 2. \$ 250 for 6 yr. at 5 % | 7. \$ 780 for 14 yr. at 6 % |
| 3. \$ 300 for 5 yr. at 6 % | 8. \$ 275 for 15 yr. at 5 % |
| 4. \$ 400 for 8 yr. at 7 % | 9. \$ 1000 for 20 yr. at 4 % |
| 5. \$ 650 for 10 yr. at 4 % | 10. \$ 50 for 20 yr. at 7 % |

Insurance

Insurance is security against damage or loss, and is of two general kinds, property and personal.

The former includes insurance against fire, flood, marine disaster, burglary, default, theft, etc. The latter includes life insurance, insurance against accident, illness, libel, etc. Of these, only property and life insurance will be discussed here.

The **policy** is the written or printed contract between the insurance company and the person insured.

The **premium** is the sum paid by the insured at stated periods, usually annually, for his insurance.

The **rate** is the specified number of cents or dollars paid for each \$100 of insurance.



A LOS ANGELES COMMERCIAL CENTER, SHOWING TITLE INSURANCE BUILDING

Property Insurance. — Property may be insured against damage by fire, flood, marine disaster, collision, and the like. In property insurance the premium is usually stated as so much per \$100, or as so many per cent.

EXAMPLE. If a house is insured for \$ 3000 for 1 year at 1 %, what is the premium ?

$$\begin{array}{r} \$ 3000 \text{ Amount of insurance} \\ \quad .01 \text{ Rate} \\ \hline \$ 30.00 \text{ Premium} \end{array}$$

Exercise

1. A merchant insures his stock of goods for \$ 4000 at 1 %. What premium does he pay ?
2. A factory worth \$ 36,000 is insured for $\frac{5}{6}$ of its value at 2 %. What is the premium ?
3. A man insured his dwelling valued at \$ 75,000 for $\frac{4}{5}$ of its value at $1\frac{1}{2}$ %. In case of total loss how much would the owner lose, including the premium paid ?
4. A dealer insured his stock of goods valued at \$ 16,000 for $\frac{7}{8}$ of their value at $1\frac{3}{4}$ %. In case of total destruction by fire what would be the owner's loss, including the premium ?
5. A professor insured his library for its full value, \$ 8000, at $1\frac{3}{4}$ %. What premium did he pay ?
6. A stock of goods worth \$ 39,000 is insured for $\frac{3}{4}$ of their value at \$.95. What is the amount of the premium ?
7. What will it cost to insure a building for \$ 3000 at $\frac{1}{2}$ % and the contents for \$ 2200 at $1\frac{1}{2}$ % for one year ?
8. A dwelling valued at \$ 15,000 was insured for $\frac{2}{3}$ of its value at $\frac{3}{4}$ % and contents valued at \$ 50,000 for $\frac{4}{5}$ of their value at 1 %. What premium was paid ?
9. If the three-year rate is twice the rate for one year, find the cost of insuring a dwelling for \$ 7200 for three years if the annual rate is $\frac{1}{2}$ %.
10. If insurance on inflammable and explosive materials is 7 % per year, how frequently can a manufacturer of such goods afford to have his plant destroyed by fire in order to save the cost of insurance ?

Personal Insurance.—There are many varieties of insurance policies. Only the simplest form will be treated here.

In life insurance the premium is usually stated as so much on \$1000 of insurance. Thus, a premium of \$26.72 means \$26.72 on each \$1000.

The rate of insurance increases as the age of the applicant increases, as indicated in the following table.

PREMIUM RATES FOR LIFE INSURANCE PER \$1000 .

AGE	ORDINARY LIFE	20-PAYMENT LIFE
20	\$ 19.21	\$ 29.44
25	21.49	31.83
30	24.38	34.76
35	28.11	38.34
40	33.01	42.79
45	39.55	48.52
50	48.48	56.17

Exercise

In the following examples use the rates given in the above table :

1. What is the annual premium on an ordinary life policy of \$2000 at the age of 25?
2. What is the annual premium on a 20-payment life policy of \$4000 at the age of 40?
3. A man at the age of 30 takes out a \$2000 ordinary life policy and dies 15 years later. What per cent of the face of the policy has he paid in premiums?
4. A man at 45 took out a \$10,000 ordinary life policy. At the age of 60 how much had he paid in premiums? What fractional part of the face of the policy had he paid?

5. The day he was 25 a man took out a 20-payment policy for \$4000; when he was 30 he took out a 20-payment policy for \$6000; when he was 40 an ordinary life policy for \$10,000. If he died when he was between 44 and 45 years old, how much more did his estate receive than he had paid for insurance?

CHAPTER VI

DENOMINATE NUMBERS

Measures of Distance

12 Inches (in.)	= 1 Foot (ft.)
3 Feet	= 1 Yard (yd.)
$5\frac{1}{2}$ Yards or $16\frac{1}{2}$ Feet	= 1 Rod (rd.)
320 Rods	= 1 Mile (mi.)
1 mi. = 320 rd. = 1760 yd. = 5280 ft.	

ORAL DRILL

1. How many inches in 3 ft.? in 5 ft.? in 7 ft.? in 10 ft.?
2. How many feet in 36 in.? in 48 in.? in 60 in.? in 100 in.?
3. How many feet in 5 yd.? in 6 yd.? in 15 yd.? in 100 yd.?
4. How many feet in 4 rd.? in 6 rd.? in 2 rd.? in 100 rd.?
5. How many yards in 2 rd.? in 4 rd.? in 6 rd.? in 100 rd.?
6. How many inches in 4 ft. 6 in.? in 3 yd. 2 ft.?
7. How many feet in 8 yd. 2 ft.? in 9 yd. 2 ft.?
8. What part of a foot is 9 in.? is 2 in.? is 3 in.? is 4 in.?
9. How many rods is $\frac{1}{8}$ of a mile? $\frac{1}{4}$ of a mile? $\frac{1}{2}$ of a mile?
10. What part of a mile is 80 rd.? 32 rd.? 64 rd.?
11. What part of a mile is 220 yd.? 440 yd.? 880 yd.?
12. At 16 cents a yard what will $2\frac{1}{4}$ yd. of ribbon cost? $5\frac{1}{2}$ yd.?

Other Measures of Distance

A *nautical mile* or *knot* is 6082.27 ft. or about 1.15 common miles.

A *league* is 3 nautical miles.

A *fathom*, used in measuring depths of water, is 6 feet.

A *cable length*, used for greater depths or distances, is 120 fathoms.

A *hand*, used in measuring heights of horses, is 4 inches.

A *furlong*, used occasionally in laying out race-courses, is 40 rods or one eighth of a mile.

Surface Measures

144 Square Inches (sq. in.)	= 1 Square Foot (sq. ft.)
9 Square Feet	= 1 Square Yard (sq. yd.)
30 $\frac{1}{4}$ Square Yards	= 1 Square Rod (sq. rd.)
160 Square Rods	= 1 Acre (A.)
640 Acres	= 1 Square Mile (sq. mi.)
1 A. = 160 sq. rd. = 4840 sq. yd. = 43,560 sq. ft.	

In roofing, flooring, painting; etc., the unit of area is 100 square feet, commonly called a *square*.

ORAL DRILL

1. How many square yards in 36 square feet? in 72 square feet?
2. How many square yards in $\frac{1}{2}$ an acre? in $\frac{1}{4}$ an acre?
3. What part of an acre is 40 square rods? 20 square rods?
4. How many square yards in 54 square feet? in 84 square feet?
5. How many acres in $\frac{1}{2}$ a square mile? in $\frac{1}{4}$ a square mile?
6. In $2\frac{2}{3}$ square yards, how many square feet?

Exercise

1. A baseball diamond is 90 feet square. What is its area in square feet? In square yards? In square rods?
2. A tennis court is 78 feet long and 36 feet wide. What is its area in square feet? In square yards? In square rods?
3. A basketball court is 70 feet long and 50 feet wide. What is its area in square feet? In square yards? In square rods?
4. A football field is 330 feet long and 160 feet wide. What is its area in square feet? In square yards? In square rods? What fractional part of an acre does it contain?
5. How many sods 16 inches square will be required to turf a lawn 170 feet long and 84 feet wide?
6. Find the cost of covering the floor of a hall 45 feet long and 30 feet wide with matting a yard wide at 70 cents a yard.
7. Find the cost of cementing the floor of a cellar 50 feet long and 28 feet wide at \$ 1.10 per square yard.
8. If a square field is 1000 feet on a side, what is its area in acres?
9. At \$ 1 a square (*i.e.* 100 square feet) what will it cost to paint the four sides of a building 50 feet long, 20 feet wide, and 15 feet high?
10. A baseball park 250 yards by 200 yards has a tight board fence around it 14 feet high. What will it cost to paint the outside of the fence at \$.50 a square?
11. If it cost \$ 320 to fence a field 80 rods long and 60 rods wide, how much will it cost to fence a field 90 rods square?
12. What will be the cost of plastering the ceiling of a room 22 feet by 18 feet at 12 cents a square yard?
13. What will be the cost of painting the outside of a house 52 feet long, 34 feet wide, and 22 feet high at 22 cents a square yard?

14. A piece of land 3 miles square can be cut up into how many 160-acre farms?

15. How much will it cost to cover with concrete a cellar floor 40 feet by 20 feet at 90 cents a square yard?

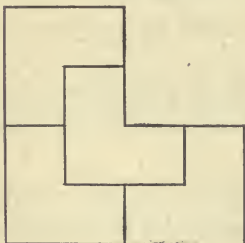
16. What will it cost to construct a 12-foot concrete sidewalk on two sides of a corner lot 40 feet by 150 feet if the sidewalk costs \$1.10 per square yard?

17. What will be the cost of a concrete sidewalk 12 feet wide around a city block 420 feet square at \$1.20 per square yard?

18. If bricks are 2 inches by 4 inches by 8 inches, what will be the cost at \$10 a thousand for enough bricks to pave a piece of sidewalk 12 feet wide, 250 feet long, if the bricks are laid flatwise?

19. At the same price what will it cost to pave the same sidewalk as in Example 18 if the bricks are laid edgewise?

20. What are the dimensions of a 40-acre rectangular field that is four times as long as it is wide?



21. The field represented in the figure was originally one mile square, but has had one quarter of it cut out of one corner. The remainder has been subdivided into four equal parts. Find the dimensions in rods of each of these parts.

Land Measures

640 Acres	= 1 Square Mile
1 Mile Square	= 1 Section
36 Square Miles	= 1 Township

An acre in the form of a square is about 209 feet on a side.

Public Lands. — A township is six miles square and contains thirty-six sections, which are numbered from the northeast corner of the township across to the west, and back on the second tier to the east, then west again on the third tier, and so on, as indicated in the accompanying diagram.



SURVEYOR'S LEVEL AND TRANSIT

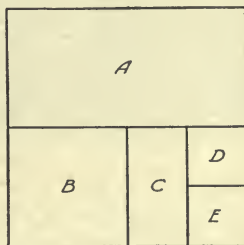
Section 16 is designated as *school land*. In some of the newer states Section 36 is also designated as *school land*.

The diagram represents a section of land and its subdivisions, which are described as follows :

A is the north half of the section, and contains 320 acres.

B is the southwest quarter of the section, and contains 160 acres.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36



C is the west half of the southeast quarter, and contains 80 acres.

D is the northeast quarter of the southeast quarter, and contains 40 acres.

E is the southeast quarter of the southeast quarter, and contains 40 acres.

Measures of Volume

1728 Cubic Inches (cu. in.) = 1 Cubic Foot (cu. ft.)

27 Cubic Feet = 1 Cubic Yard (cu. yd.)

128 Cubic Feet = 1 Cord (cd.)

A *load* of earth, sand, concrete, or similar material is considered to be a cubic yard.

Cubic or *solid* measure is used in computing the volumes of solids, and the capacity of bins, boxes, etc.

ORAL DRILL

1. How many cubic feet in 2 cubic yards? in 3 cubic yards? in 5 cubic yards?
2. How many cubic inches in 2 cubic feet? in 10 cubic feet?
3. What part of a cubic yard are 9 cubic feet? 12 cubic feet?
4. How many cubic feet in 2 cords? in $\frac{1}{2}$ cord? in $\frac{1}{4}$ cord?
5. What part of a cubic yard is 18 cubic feet? 21 cubic feet?
6. What part of a cubic foot is 1 cubic inch? 864 cubic inches?
7. What part of a cubic yard is 1 cubic foot? $13\frac{1}{2}$ cubic feet?
8. What part of a cubic foot is a 6-inch cube?
9. What part of a cubic yard is an 18-inch cube?
10. What part of a cubic foot is a 4-inch cube?

Rule.—To find the volume or capacity, as of a cellar, multiply together the length, width, and depth.

EXAMPLE. Find the cost at 25 cents per cubic yard of excavating a cellar 36 feet in length, 21 feet in width, and 8 feet deep.

$$\begin{array}{r} 4 \qquad 7 \\ 36 \times 21 \times 8 \\ \hline 21 \end{array} = 224, \text{ the number of cubic yards}$$

$$$.25 \times 224 = \$56.00, \text{ the cost}$$

Exercise

1. Find the cost of excavating a cellar 32 feet long, 18 feet wide, and 6 feet deep at 25 cents per cubic yard.
2. Twenty-nine pupils and their teacher occupy a school-room 32 feet in length, 24 feet in width, and 14 feet in height. What is the average number of cubic feet of air for each person?



A STEAM DREDGER

3. A city lot 50 feet by 150 feet is to be covered with a layer of earth one foot deep except where the house stands. The house is 36 feet by 54 feet. Find the number of loads of earth required, and its cost at \$.25 a load.
4. How many bushels of grain will a bin hold that is 16 feet long, 6 feet wide, and 4 feet deep, if we allow $1\frac{1}{4}$ cubic feet to the bushel?

Measures of Capacity

Liquid

4 Gills (gi.)	= 1 Pint (pt.)
2 Pints	= 1 Quart (qt.)
4 Quarts	= 1 Gallon (gal.)
$31\frac{1}{2}$ Gallons	= 1 Barrel (bbl.)
63 Gallons	= 1 Hogshead (hhd.)
1 gal. = 4 qt. = 8 pt. = 32 gills	

A *gallon* contains about 231 cubic inches. A gallon of water weighs about 8.3 pounds.

A pail six or seven inches in diameter and the same depth contains about a gallon.

A *barrel* contains about 4.2 cubic feet.

A *cubic foot* of water contains about 7.5 gallons and weighs about 62.3 pounds, or 1000 ounces.

ORAL DRILL

1. How many gills in 3 pints? in $3\frac{1}{2}$ pints? in 2 quarts?
2. How many pints in 1 gallon? in $2\frac{1}{2}$ gallons? in 3 gallons?
3. In 2 gallons how many quarts? pints? gills?
4. In 32 pints how many quarts? how many gallons?
5. What part of a quart are 6 gills? What part of a gallon?
6. What part of 2 gallons are 4 pints? are 8 pints? 2 quarts?
7. At 8 cents a pint what will 4 quarts of milk cost?
8. If a gallon of gasoline costs 21 cents, what will 5 gallons cost?
9. How many pint bottles will be required to hold 5 gallons of gasoline?
10. What is the profit if gasoline is bought for 20 cents a gallon and sold for 10 cents a pint?

Measures of Capacity**Dry**

2 Pints (pt.) = 1 Quart (qt.)

8 Quarts = 1 Peck (pk.)

4 Pecks = 1 Bushel (bu.)

1 bu. = 4 pk. = 32 qt. = 64 pt.

Dry measure is used in measuring such substances as grain, seeds, vegetables, feed, etc.



13,000,000 BUSHEL COAL SHIPMENT BY BARGES

The *Winchester bushel* is the standard measure for dry substances. It is a cylindrical vessel $18\frac{1}{2}$ inches in diameter and 8 inches deep, and contains 2150.42 cubic inches.

A bushel contains about $1\frac{1}{4}$ cubic feet. A cubic foot is $\frac{4}{5}$ of a bushel.

ORAL DRILL

1. How many quarts in 3 pecks? in 6 pecks? in a bushel and a half?

2. In 96 quarts how many pecks? how many bushels?
3. How many quart boxes will a bushel and a half fill?
4. At 5 cents a pint what is a bushel of peanuts worth?
5. At 20 cents a quart what will 2 bushels of walnuts cost?
6. If peanuts are bought for \$2 a bushel and sold at 5 cents a pint, what is the gain?
7. How many bushels in 5 cubic feet?
8. Which has the greater volume, a gallon or a bushel?
9. Which has the greater volume, a cubic foot or a gallon?
10. Which has the greater volume, a bushel or a barrel?

Exercise

1. A bin is 8 feet long, 6 feet wide, and 4 feet deep. Estimate roughly its capacity in bushels. Compute somewhat more closely its capacity.

2. A tank is 8 feet long, 6 feet wide, and 4 feet deep. Estimate roughly its capacity in gallons. Compute its capacity to a greater degree of accuracy.

3. A tank is 12 feet long and 6 feet wide. How deep must it be to hold 2500 gallons?

4. A man had 42 gallons of vinegar. He sold 6 gallons to one person, a gallon and a half to another, and 15 gallons to a third. What is the remainder worth at 10 cents a quart?

5. Five gallons of liquid will fill how many half-pint bottles?

6. How many 2-ounce bottles can be filled with 5 gallons of vanilla extract?

7. If a milkman retails on an average 100 quarts of milk a day, how many gallons of milk will he sell in three months?

8. A spice merchant put up 2 tons of ground mustard in one-quarter-pound cans. If the mustard cost him \$330 a ton, the cans half a cent each, and the labor of canning \$.80 per

hundred cans, what was his profit if he sold the mustard at 10 cents a can?

9. A box filled with sand is 16 feet long, 10 feet wide, and 16 inches deep. If sand weighs 165 pounds to the cubic foot,



GIANT ORE CRANE LIFTING OVER A TON OF ORE

what is the weight of the contents of the box? Give the answer in tons.

10. A rainfall of one inch amounts to how many gallons of water to the acre? to how many tons?

11. If the rainfall on a certain day in Virginia City, Nevada, was 2.25 inches, find the number of cubic feet that fell on a lot 209 feet square. Find the number of gallons.

Measures of Weight

Avoirdupois

$$\begin{aligned} 16 \text{ Ounces (oz.)} &= 1 \text{ Pound (lb.)} \\ 2000 \text{ Pounds} &= 1 \text{ Ton (T.)} \end{aligned}$$

2240 pounds is called a *long ton*. It is used in the custom house, and where coal and ore are handled.

Avoirdupois weight is used for most commodities in common use except gold and silver.

Troy

$$\begin{aligned} 24 \text{ Grains (gr.)} &= 1 \text{ Pennyweight (pwt.)} \\ 20 \text{ Pennyweights} &= 1 \text{ Ounce (oz.)} \\ 12 \text{ Ounces} &= 1 \text{ Pound (lb.)} \\ 1 \text{ lb.} &= 12 \text{ oz.} = 240 \text{ pwt.} = 5760 \text{ gr.} \end{aligned}$$

A *carat* is about 3.2 Troy grains, and is used in weighing diamonds.

Troy weight is used for weighing gold, silver, diamonds, and other precious minerals.

The avoirdupois pound contains 7000 grains; the Troy pound, 5760 grains.

Apothecaries

$$\begin{aligned} 20 \text{ Grains (gr.)} &= 1 \text{ Scruple (sc.)} \\ 3 \text{ Scruples} &= 1 \text{ Dram (dr.)} \\ 8 \text{ Drams} &= 1 \text{ Ounce (oz.)} \\ 12 \text{ Ounces} &= 1 \text{ Pound (lb.)} \end{aligned}$$

Apothecaries' weight is used by druggists and physicians in compounding and prescribing medicines.

Measures of Time

60 Seconds (sec.)	= 1 Minute (min.)
60 Minutes	= 1 Hour (hr.)
24 Hours	= 1 Day (da.)
7 Days	= 1 Week (wk.)
365 Days	= 1 Common Year (yr.)
366 Days	= 1 Leap Year
100 Years	= 1 Century (Cent.)

Circular Measures

60 Seconds (")	= 1 Minute (')
60 Minutes	= 1 Degree (°)
360 Degrees	= 1 Circumference (circ.)

This table is used in measuring angles, and in reckoning latitude and longitude.

Difference in Time

Difference in time is the interval between two specified dates.

EXAMPLE 1. Find the difference in time between March 6, 1904, and August 15, 1916.

YEAR	MONTH	DAY	August 15, 1916, is set down as the 15th day of the 8th month of the year 1916. Similarly March 6, 1904, is set down as the 6th day of the 3d month of the year 1904.
1916	8	15	
1904	3	6	
12	5	9	

By subtraction, the difference in time is seen to be 12 years, 5 months, and 9 days.

EXAMPLE 2. Find the difference in time between Dec. 11, 1900, and June 24, 1916.

YEAR	MONTH	DAY
1915	18	
~~1916~~	~~8~~	24
1900	12	11
15	6	13

As the number of the month in the subtrahend is greater than that in the minuend, diminish the number of the year by 1 and increase the number of the month by 12. Subtract as before and the difference is seen to be 15 years, 6 months, and 13 days.

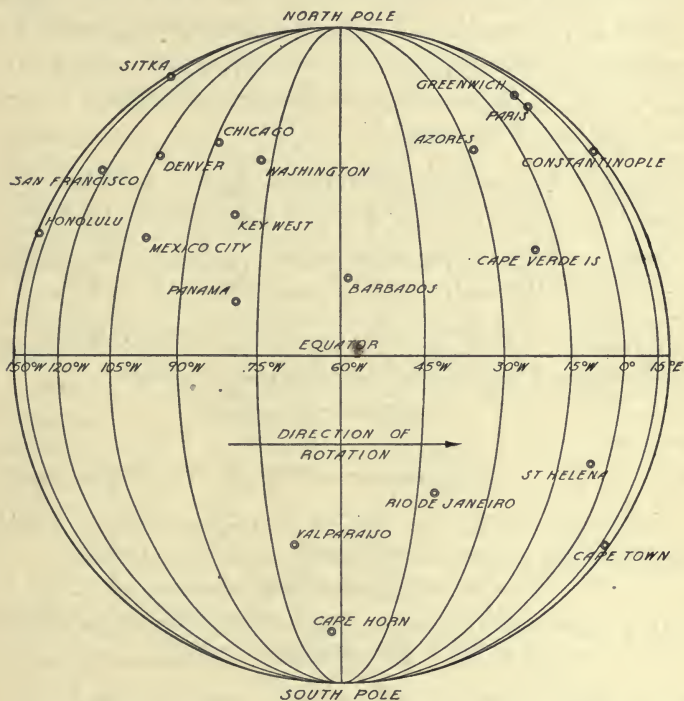
Exercise

1. How many days from May Day to Christmas?
2. How many days from Memorial Day to Independence Day?
3. How many days from Lincoln's Birthday to Independence Day?
4. How many days from Washington's Birthday to Thanksgiving Day in the current year?
5. How many days from the first day of spring to the first day of winter?
6. How many days from Christmas to the Fourth of July?
7. Find the difference in time between Nov. 23, 1875, and Dec. 11, 1900.
8. Find the time interval between Oct. 10, 1845, and March 6, 1904.
9. Find the time between June 11, 1840, and Dec. 8, 1870.
10. If the Panama Pacific Exposition opened Feb. 20, 1915, and closed Dec. 4 of the same year, how long was it open?

Longitude and Time

The *longitude* of a place is its distance in degrees east or west of an established meridian, sometimes called the *prime meridian*. The meridian through Greenwich, England, is ordinarily used as the prime meridian.

Rule. — To find the difference in longitude between two places, subtract if both are east or both are west of an established meridian ; if one is east and the other west, add.



EXAMPLE 1. Find the difference in longitude between Boston and San Francisco.

The longitude of San Francisco is . . . 122° 25' west

The longitude of Boston is . . . 71° 4' west

Their difference is . . . 51° 21'

EXAMPLE 2. Find the difference in longitude between Constantinople and Sitka.

The longitude of Constantinople is . . .	28° 59' east
The longitude of Sitka is	135° 20' west
The sum is	164° 19'
The difference in longitude, then, is . .	164° 19'

Due to the rotation of the earth and the division of the equator into 360 degrees, the sun apparently passes over $\frac{1}{24}$ of 360° or 15° each hour. Similarly it appears to pass over 1° every 4 minutes.

Rule. — To find the difference in time in minutes, multiply by 4 the number of degrees difference in longitude.

Rule. — To find the difference in longitude in degrees, divide by 4 the number of minutes difference in time.

EXAMPLE 1. Find the difference in time between Boston and San Francisco if the difference in longitude is 51 degrees.

$$\begin{array}{r} 51, \text{ difference in longitude} \\ \underline{4} \\ 204 \text{ minutes, or 3 hr. 24 min., difference in time} \end{array}$$

EXAMPLE 2. Find the difference in longitude between London and New York if there is a difference in time of about 5 hours.

$$\begin{array}{r} 5 \text{ hours is 300 minutes} \\ 75, \text{ the number of degrees difference in longitude} \\ 4 \overline{)300} \text{ minutes difference in time} \end{array}$$

Exercise

1. Through how many degrees does the earth turn in 1 hour? in 5 hours?
2. When it is noon at Washington, what time is it at a place 15° 30' east of Washington? 45° 45' west?
3. What is the difference in longitude between two places whose difference in time is 1 hour, 10 minutes?

4. If a person travels until his watch is 2 hours fast, in what direction and through how many degrees has he traveled?

5. What is the difference in time between two places whose longitudes are 75° east and 60° east? 75° east and 60° west? 75° west and 60° west?

LONGITUDES OF VARIOUS CITIES

Antwerp . . .	$4^{\circ} 25' \text{ E.}$	Naples . . .	$14^{\circ} 14' \text{ E.}$
Berlin . . .	$13^{\circ} 24' \text{ E.}$	New York . .	$74^{\circ} 00' \text{ W.}$
Boston . . .	$71^{\circ} 04' \text{ W.}$	Panama . . .	$79^{\circ} 32' \text{ W.}$
Buenos Aires . .	$58^{\circ} 22' \text{ W.}$	Paris . . .	$2^{\circ} 20' \text{ E.}$
Capetown . . .	$18^{\circ} 29' \text{ E.}$	Pittsburgh . .	$80^{\circ} 03' \text{ W.}$
Chicago . . .	$87^{\circ} 37' \text{ W.}$	Rome . . .	$12^{\circ} 29' \text{ E.}$
Constantinople .	$28^{\circ} 59' \text{ E.}$	San Francisco .	$122^{\circ} 25' \text{ W.}$
Denver . . .	$104^{\circ} 59' \text{ W.}$	St. Louis . . .	$90^{\circ} 12' \text{ W.}$
Havana . . .	$82^{\circ} 22' \text{ W.}$	Sydney . . .	$151^{\circ} 12' \text{ E.}$
Honolulu . . .	$157^{\circ} 52' \text{ W.}$	Tokyo . . .	$139^{\circ} 44' \text{ E.}$
Manila . . .	$120^{\circ} 58' \text{ E.}$	Valparaiso . .	$71^{\circ} 39' \text{ W.}$
Mexico City . .	$99^{\circ} 07' \text{ W.}$	Washington . .	$77^{\circ} 03' \text{ W.}$

Exercise

Using the above table answer the following:

1. When it is noon at San Francisco what time is it at Antwerp? at Berlin? at Boston? at Buenos Aires?

2. When it is noon at Constantinople what time is it at Capetown? at Chicago? at Denver? at Havana?

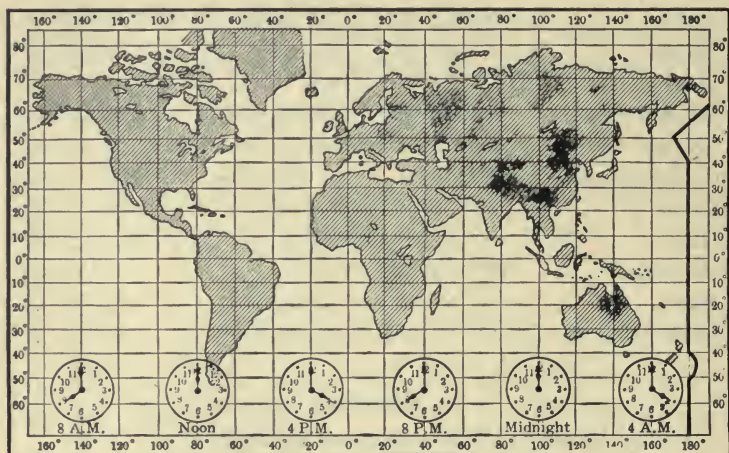
3. When it is 6 A.M. at New York what time is it at Honolulu? at Manila? at Mexico City? at Naples?

4. When it is 3 P.M. at Panama what time is it at Paris? at Pittsburgh? at Rome? at San Francisco?

5. When it is midnight at Washington what time is it at St. Louis? at Sydney? at Tokyo? at Valparaiso?

Standard Time

For practical purposes a great part of the civilized world is now divided into sections, all places in the same section using the time of a certain meridian in that section. To make the times of adjacent sections just one hour apart, the meridian usually chosen is 15° , 30° , 45° , or other multiple of 15° , east or west of the meridian



MAP SHOWING TIME OF DIFFERENT LOCALITIES AT A DEFINITE MOMENT

through Greenwich. Thus, England uses the time of Greenwich, that is, 0° ; Central Europe uses that of 15° E.; Japan uses that of 135° E.; and the Pacific Slope of the United States that of 120° W.

A time system where all places in a section of territory near some chosen meridian have the same time is called a system of *standard time*. The standard time map of the United States is shown on the following page.

The irregular boundaries of the sections are due to a

desire to have the division lines pass through important railway centers.

When it is noon by Eastern time it is 11 A.M. by Central time.

When it is noon by Eastern time it is 10 A.M. by Mountain time.

When it is noon by Eastern time it is 9 A.M. by Pacific time.



STANDARD TIME MAP OF THE UNITED STATES

Exercise

In the following examples use the above map.

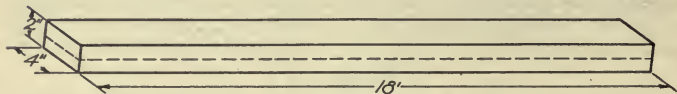
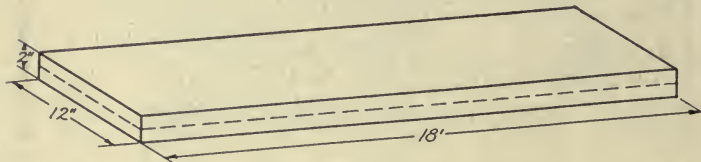
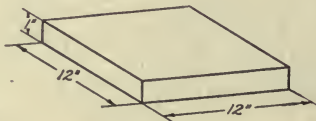
1. When it is noon in California what time is it in Vermont? in Kentucky? in Wyoming? in eastern Arizona?
2. When it is 6 P.M. in Massachusetts what time is it in Michigan? in Oklahoma? in Utah? in Florida?
3. When it is midnight in Iowa what time is it in Montana? in Washington? in Ohio? in New York?
4. When it is 6 A.M. in Maine what time is it in Alabama? in Minnesota? in Oregon? in Virginia?

5. When it is 9:30 A.M. in Colorado what time is it in Florida? in Pennsylvania? in California? in Texas?

6. An order from San Francisco to Boston is wired at 2 P.M. The wire takes $\frac{3}{4}$ hr. to reach Boston, where the office of the recipient closes at 6. Will the telegram be received before closing time?

Board and Lumber Measure

The *board foot* is the unit used in measuring lumber, and is the amount of lumber in a piece one foot wide, one inch thick, and one foot long.



Rule.—To find the number of board feet in a piece of lumber, multiply $\frac{1}{12}$ of the number of square inches in an end section of the piece by the number of feet in the length.

EXAMPLE 1. Find the number of board feet in a piece of lumber 1 foot wide, 2 inches thick, and 18 feet long.

$$\frac{2 \times 12}{12} \times 18 = 36, \text{ the number of board feet in the piece.}$$

EXAMPLE 2. Find the number of board feet in a scantling 2 inches by 4 inches, 18 feet long.

$$\frac{2 \times 4}{12} \times 18 = 12, \text{ the number of board feet in the piece.}$$

ORAL DRILL

1. How many board feet are there in a board 1 foot wide, 1 inch thick, and 12 feet long? 14 feet long? 8 feet long?



BIG TREES

2. How many board feet are there in a board 6 inches wide, 1 inch thick, and 12 feet long? 16 feet long? 18 feet long?

3. How many board feet are there in a piece 16 feet long, 1 foot wide, and 4 inches thick? 6 inches thick? 8 inches thick?

Find the number of board feet in each of the following pieces, 1 inch thick, of length and width as follows :

- | | | |
|------------------|-------------------|-------------------|
| 4. 18 ft., 4 in. | 7. 16 ft., 9 in. | 10. 18 ft., 9 in. |
| 5. 16 ft., 6 in. | 8. 18 ft., 18 in. | 11. 18 ft., 3 in. |
| 6. 12 ft., 8 in. | 9. 12 ft., 12 in. | 12. 14 ft., 4 in. |

Exercise

Find the number of board feet in the following:

1. 8 planks 2 in. thick, 10 ft. long, 12 in. wide.
2. 12 planks 3 in. thick, 14 ft. long, 10 in. wide.
3. 16 planks $2\frac{1}{2}$ in. thick, 16 ft. long, 10 in. wide.
4. 16 planks $1\frac{3}{4}$ in. thick, 10 ft. long, 9 in. wide.
5. 2 sticks of timber 18 ft. long, 1 ft. wide, 8 in. thick.
6. 5 sticks of timber 16 ft. long, 10 in. wide, 9 in. thick.
7. 25 joists 3" by 4" and 16 ft. long.
8. 30 joists 2" by 4" and 14 ft. long.
9. 50 joists 4" by 8" and 16 ft. long.
10. 8 beams 12" by 12" and 12 ft. long.

Lumber less than 1 inch in thickness is figured as if it were 1 inch thick. Material over 1 inch thick is figured according to its actual thickness in inches and fractions of an inch.

Lumber is usually priced at so much *per thousand* (M) board feet. In practice the cost is computed at so much per board foot. Thus, lumber at \$20 per thousand feet (M) is \$.02 per board foot.

ORAL DRILL

Find the cost in each of the following:

1. 2000 ft. of Oregon pine at \$22 per M.
2. 5000 ft. of pine flooring at \$24 per M.
3. 10,000 ft. of rough redwood at \$21 per M.
4. 20,000 ft. of spruce at \$27 per M.
5. 6000 ft. of oak at \$50 per M.

Exercise

Find the cost in each of the following :

1. 30 boards 1 in. thick, 8 in. wide, and 12 ft. long, at \$ 22 per M.
2. 80 boards 1 in. thick, 8 in. wide, and 14 ft. long, at \$ 25 per M.
3. 50 boards $\frac{7}{8}$ in. thick, 10 in. wide, and 16 ft. long, at \$ 24 per M.
4. 100 planks 2 in. thick, 10 in. wide, and 18 ft. long, at \$ 26 per M.
5. 200 boards $\frac{3}{4}$ in. thick, 6 in. wide, and 16 ft. long, at \$ 30 per M.
6. 8 pieces 2" by 6", 12 ft. long, at \$ 24 per M.
7. 24 pieces 2" by 8", 16 ft. long, at \$ 22.50 per M.
8. 6 pieces 8" by 10", 18 ft. long, at \$ 27 per M.
9. 3 pieces 40 ft. long, and 10" by 12", at \$ 40 per M.
10. 30 pieces 16 ft. long, and 2" by 3", at \$ 22 per M.

Measures of Value

United States Money

10 Mills (m.)	= 1 Cent (c.)
10 Cents	= 1 Dime (d.)
10 Dimes	= 1 Dollar (\$)
10 Dollars	= 1 Eagle (E.)

English Money

4 Farthings (f.)	= 1 Penny (1 d.)
12 Pence	= 1 Shilling (1 s.)
20 Shillings	= 1 Pound (£ 1)
£ 1	= \$ 4.87



DENVER MINT

PRE-WAR VALUES IN UNITED STATES MONEY OF CERTAIN
FOREIGN COINS

Franc	France, Belgium, and Switzerland . .	\$.193
Lira	Italy	.193
Peseta	Spain	.193
Crown	Austria-Hungary	.203
Mark	Germany	.238
Crown	Sweden	.268
Peso	Chile	.365
Yen	Japan	.498
Ruble	Russia	.515

Miscellaneous Measures

Paper	Counting
24 Sheets = 1 Quire (qr.)	12 Things = 1 Dozen (doz.)
20 Quires = 1 Ream (rm.)	12 Dozen = 1 Gross (gro.)
2 Reams = 1 Bundle (bdl.)	12 Gross = 1 Great Gross
5 Bundles = 1 Bale (bl.)	20 Things = 1 Score

The Metric System

The *Metric System* is a system of weights and measures which originated in France. It is a decimal system of great simplicity and has been adopted in nearly all civilized countries except the United States and the British Empire. It is almost universally used by scientists.

The primary units are the *meter*, *liter*, and *gram*. All other units are named from the primary units by using Latin and Greek prefixes. The Latin prefixes denote *parts* of the primary unit; the Greek prefixes denote *multiples* of the primary unit.

Latin Prefixes

Milli means $\frac{1}{1000}$; thus, *millimeter* means $\frac{1}{1000}$ of a meter.

Centi means $\frac{1}{100}$; thus, *centimeter* means $\frac{1}{100}$ of a meter.

Deci means $\frac{1}{10}$; thus, *decimeter* means $\frac{1}{10}$ of a meter.

Greek Prefixes

Deca means 10; thus, *decameter* means 10 meters.

Hecto means 100; thus, *hectometer* means 100 meters.

Kilo means 1000; thus, *kilometer* means 1000 meters.

Myria means 10,000; thus, *myriameter* means 10,000 meters.

Length

In the metric system the standard *unit of length* is the *meter*. It is about 39.37 inches. From the meter all the other units in the system are derived.

A *decimeter*, or one tenth of a meter, is about 4 inches.

A *centimeter*, or one one-hundredth of a meter, is about $\frac{2}{5}$ of an inch.

A *millimeter*, or one one-thousandth of a meter, is about $\frac{1}{25}$ of an inch.

The above-mentioned three units are used in the measurement of short distances. For greater distances a larger unit is desirable. For such purposes the *kilometer*, or 1000 meters, is used. It is about $\frac{5}{8}$ of a mile.

Weight

The *unit of weight* is the *gram*, which is the weight of a cubic centimeter of pure water at 4° Centigrade.

A *decigram* is one tenth of a gram.

A *centigram* is one one-hundredth of a gram.

A *milligram* is one one-thousandth of a gram.

These units are used in weighing small bodies. For heavier bodies a larger unit is used, namely, the *kilogram*, which is 1000 grams.

The kilogram is about 2.2 pounds.

Volume

The *unit of volume* is the *liter*, or cubic decimeter, and is equal to about $2\frac{1}{5}$ pints.

Summary

The metric units are subdivided decimally. Thus, to express .1 of a meter, .1 of a gram, and .1 of a liter, the Latin prefix *deci-* is used in each case. *Decimeter* means $\frac{1}{10}$ of a meter; *decigram* $\frac{1}{10}$ of a gram; and *deciliter* $\frac{1}{10}$ of a liter.

To express .01 of a meter, gram, and liter, the Latin prefix *centi-* is used. Thus, *centimeter* means $\frac{1}{100}$ of a meter; *centigram* means $\frac{1}{100}$ of a gram, etc.

To express .001 of a meter, gram, and liter, the Latin prefix *milli-* is used, giving rise to the terms *millimeter*, *milligram*, etc.

As the system is a decimal one, 10 times one unit of any denomination equals one unit of the next higher denomination.

To express 10 times a meter, 10 times a gram, and 10 times a liter, the Greek prefix *deca-* is used in each case, so *decameter* means 10 meters, *decagram* means 10 grams, and *decaliter* means 10 liters.

To express 100 times a meter, gram, and liter, the Greek prefix *hecto-* is used in each case. Thus *hectometer* means 100 meters; *hectogram* means 100 grams, etc.

To express 1000 times a meter, gram, and liter, the Greek prefix *kilo-* is used, giving rise to the terms *kilometer*, *kilogram*, etc.

Metric Conversion Tables

Length

TO CONVERT	MULTIPLY BY	TO CONVERT	MULTIPLY BY
Miles to kilometers .	1.60935	Kilometers to miles .	0.62137
Miles to meters .	1609.347	Meters to miles . .	0.0006214
Yards to meters . .	0.91440	Meters to yards . .	1.09361
Feet to meters . . .	0.30480	Meters to feet . . .	3.28083
Inches to centimeters .	2.54000	Centimeters to inches	0.39370
Inches to millimeters .	25.40005	Millimeters to inches	0.03937

Area

Square yards to sq.		Sq. meters to sq. yd. .	1.19599
meters (m. ²) . .	0.83613	Sq. meters to sq. ft. .	10.76387
Sq. ft. to sq. meters (m. ²)	0.09290	Sq. cm. to sq. in. . .	0.15500
Sq. in. to sq. cm. (cm. ²)	6.45163	Sq. mm. to sq. in. .	0.00155
Sq. in. to sq. mm.			
(mm. ²)	645.163		

Volume

TO CONVERT	MULTIPLY BY	TO CONVERT	MULTIPLY BY
Cubic yards to cu. meters (m. ³) . .	0.76456	Cu. meters to cu. yd.	1.30802
Cu. ft. to cu. meters (m. ³)	0.02832	Cu. meters to cu. ft. .	35.31661
Cu. in. to cu. cm. (cm. ³)	16.38716	Cu. cm. to cu. in. .	0.06102
Cu. ft. to liters . .	28.31701	Liters to cu. ft. . .	0.03532
Cu. in. to liters . .	0.01639	Liters to cu. in. . .	61.02337
		Liters to gallons . .	0.26417
		Gallons to liters . .	3.78543

Weight

Tons to kilograms .	907.18486	Kilograms to tons .	0.001102
Pounds to kilograms	0.45359	Kilograms to pounds	2.20462
Ounces to grams .	28.34953	Grams to ounces .	0.03527
Grains to grams .	0.64799	Grams to grains .	15.43236

Roman Numerals

Roman numerals are now seldom used except in date inscriptions, in numbering chapters, and upon the dials of timepieces.

The letters used and some of their combinations are as follows:

1 I	10 X	100 C	1000 M
2 II	20 XX	200 CC	2000 MM
3 III	30 XXX	300 CCC	3000 MMM
4 IV, or IIII	40 XL	400 CD	4000 $\overline{\text{IV}}$
5 V	50 L	500 D	5000 $\overline{\text{V}}$
6 VI	60 LX	600 DC	6000 $\overline{\text{VI}}$
7 VII	70 LXX	700 DCC	1620 MDCXX
8 VIII	80 LXXX	800 DCCC	1793 MDCCXCIII
9 IX	90 XC	900 CM	1915 MCMXV

The letters used are I, V, X, L, C, D, M.

When a letter of less value is written after a letter of greater value its value is added to that of the letter of greater value, as in VI, LX, DC.

When a letter of less value is written before a letter of greater value its value is taken from that of the letter of greater value, as in IX, XC, CM.

A straight line placed horizontally over a letter or combination of letters has the effect of multiplying by one thousand.

ORAL DRILL

Read the following numbers :

- | | | | |
|----------|----------|-------------|-------------|
| 1. XVI | 6. XL | 11. LXXXIII | 16. XC |
| 2. XVIII | 7. XLII | 12. LXI | 17. XCIII |
| 3. XXIV | 8. XLVII | 13. LIX | 18. MDCC |
| 4. XXIX | 9. LX | 14. LXXIX | 19. MDCCCXX |
| 5. XIX | 10. LXIX | 15. CXX | 20. MCMXV |

Write the following numbers with Roman numerals :

- | | | | | |
|--------|--------|---------|----------|----------|
| 21. 27 | 25. 59 | 29. 79 | 33. 346 | 37. 1888 |
| 22. 36 | 26. 62 | 30. 83 | 34. 458 | 38. 1918 |
| 23. 47 | 27. 67 | 31. 127 | 35. 572 | 39. 1958 |
| 24. 52 | 28. 77 | 32. 222 | 36. 1776 | 40. 2000 |

Exercise

1. How many steps does a man take in walking a mile if he advances 30 inches at each step?

2. If a man takes 37 steps for every 100 feet that he advances, what is the length of each step?

3. How many bushels will a bin 8 feet square and 6 feet deep hold?

4. How many bags of two bushels and a half capacity can be filled from 260 bushels?

5. How many gallons will a tank 4 feet 8 inches by 2 feet 10 inches by 1 foot 6 inches hold?

6. The bottom of a rectangular tank is 6 feet square. How deep must it be to hold 1000 gallons?

7. How many cakes of soap 4 inches by 3 inches by $1\frac{1}{2}$ inches can be packed in a box whose inside dimensions are 2 feet, 3 feet, and 4 feet?

8. How many cubic yards of earth must be removed in digging a cellar 36 feet long, 26 feet wide, and 8 feet deep?

9. Find the cost of digging a cellar 42 feet long, 28 feet wide, and 6 feet 6 inches deep, at 50 cents a cubic yard.

10. What is the capacity in gallons of a rectangular tank 9 feet square and 8 feet deep?

11. At \$1.20 a bushel what is the value of a bin of wheat 16 feet long, 8 feet wide, and 4 feet deep?

12. What is the capacity in gallons of a rectangular tank 12 feet long, 10 feet wide, and 6 feet deep?

13. What must be the width of a lot 200 feet long if its area is one half an acre?

14. If sound travels at the rate of 1100 feet a second, how far distant is a thunderstorm if the thunder-clap is heard 7 seconds after the lightning flash is seen?

15. If William Cullen Bryant was born Nov. 3, 1794, how old was he June 12, 1878?

16. How many years have elapsed since Dec. 22, 1620?

17. If a rug 9 feet by 12 feet is laid on the floor of a room which is 11 feet by 14 feet, how many square feet of floor surface are bare?

18. An inch of rain on a 12-acre ranch amounts to how many gallons of water?

19. What must be the length of a pile of wood 6 feet high and 4 feet wide to contain 50 cords?

20. If a ton of coal occupies 36 cubic feet, what must be the depth of a bin 8 feet wide and 10 feet long in order that it may hold 10 tons?



BRIDGE CONSTRUCTION WORK, FIRTH OF FORTH

21. The bridge over the Firth of Forth is 8296 feet long. What is its length in meters?

CHAPTER VII

THE USE OF TABLES

There are certain calculations which occur so frequently that to save time and labor their results have been tabulated. For example, the circumferences and areas of circles of various diameters, the square roots of numbers, the decimal equivalents of common fractions, etc., have all been arranged in tabular form in much the same way that multiplication tables are arranged, or as railroad time tables are arranged. These tables are not ordinarily memorized as the multiplication table is, or as tables of weights and measures are, but are consulted or referred to each time need arises for a particular result.

For example, if one wants the square root of 20, he does not actually extract the square root but refers to a table of square roots and finds it to be 4.4721. Or if he desires to know the area of a circle whose diameter is 50 he refers to a table of areas of circles and finds it to be 1963.5. Or again, if he desires to know the interest on \$ 350 for two years and six months at eight per cent, he refers to an interest table and reads his answer instantly, \$ 70.

Just what tables one needs most, depends upon one's occupation. The machinist has use for tables of decimal equivalents of common fractions, tables of cutting speeds, etc. Draughtsmen and designers use tables of strengths and weights of various materials, standard proportions of machine parts, etc. The actuary uses mortality tables, the banker interest tables, the merchant discount tables,

the electrician wiring tables, etc. The professional computer has need for all of these and many more.

There are certain mathematical tables, however, which are of general value whatever one's occupation. Among these are tables of squares, cubes, square roots, cube roots, decimal equivalents of fractions, and logarithms of numbers. The table in most common use is the multiplication table given below.

MULTIPLICATION TABLE

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Exercise

Prepare a multiplication table similar to the above for numbers from 13 to 25.

TABLE OF POST-OFFICE MONEY ORDER RATES

For Orders From \$	0.01 to \$	2.50	. . .	3 cents
	From \$	2.51 to \$	5.00	. . . 5 cents
	From \$	5.01 to \$	10.00	. . . 8 cents
	From \$	10.01 to \$	20.00	. . . 10 cents
	From \$	20.01 to \$	30.00	. . . 12 cents
	From \$	30.01 to \$	40.00	. . . 15 cents
	From \$	40.01 to \$	50.00	. . . 18 cents
	From \$	50.01 to \$	60.00	. . . 20 cents
	From \$	60.01 to \$	75.00	. . . 25 cents
	From \$	75.01 to \$	100.00	. . . 30 cents

ORAL DRILL

1. Read from the above table the charge for a post-office money order for \$ 7.50.
2. What is the charge for a \$ 12 money order?
3. What will it cost to send a money order for \$ 27.50?
4. What is the total amount, including the money order charges, for the following amounts: \$4.15, \$37.50, and \$52.00?
5. A person buys three money orders as follows: one for \$7.20, one for \$1.75, and one for \$9.50. He buys also half a dollar's worth of postage stamps. What change should he get if he tenders in payment a twenty-dollar gold piece?

SPEED SCHEDULE

TIME TO GO ONE MILE	RATE IN MILES PER HOUR	TIME TO GO ONE MILE	RATE IN MILES PER HOUR	TIME TO GO ONE MILE	RATE IN MILES PER HOUR
<i>Min. Sec.</i>		<i>Min. Sec.</i>		<i>Min. Sec.</i>	
5 00	12.00	1 22	43.90	54	66.66
4 00	15.00	1 20	45.00	53	67.92
3 00	20.00	1 18	46.15	52	69.23
2 50	21.18	1 16	47.37	51	70.59
2 40	22.50	1 15	48.00	50	72.00
2 30	24.00	1 14	48.65	49	73.47
2 24	25.00	1 13	49.31	48	75.00
2 20	25.72	1 12	50.00	47	76.59
2 15	26.67	1 11	50.70	46	78.26
2 10	27.69	1 10	51.43	45	80.00
2 5	28.80	1 9	52.17	44	81.82
2 00	30.00	1 8	52.94	43	83.72
1 55	31.30	1 7	53.73	42	85.71
1 50	32.74	1 6	54.55	41	87.80
1 45	34.29	1 5	55.38	40	90.00
1 42	35.29	1 4	56.25	39	92.31
1 40	36.00	1 3	57.14	38	94.74
1 38	36.73	1 2	58.06	37	97.30
1 36	37.50	1 1	59.02	36	100.00
1 34	38.29	1 0	60.00	35	102.86
1 32	39.13	59	61.02	34	105.88
1 30	40.00	58	62.07	33	109.09
1 28	40.91	57	63.14	32	112.50
1 26	41.86	56	64.29	31	116.13
1 24	42.86	55	65.45	30	120.00

ORAL DRILL

1. If a vehicle is traveling at the rate of a mile in a minute, read from the above table its rate in miles per hour.
2. From the table read the rate in miles per hour if a mile is traveled in 48 seconds.

3. If the speed of a train is 50 miles an hour, what is the time it takes in going one mile?
4. If the speed of an auto is 55 miles an hour, what is the time it takes in going two miles?
5. If the time for one mile is 36 seconds, what is the speed per hour?

Exercise

1. In Lieutenant Murray's record flight on June 25, 1919, from Atlantic City to New York, he covered a distance of 140 miles in 61 minutes. What was his time per mile?
2. When Lieutenant Sharpnack flew on July 2, 1919, from San Diego to San Francisco, a distance of 610 miles in 4 hours, 32 minutes, what was his rate in miles per hour?
3. On September 18, 1919, when Rohlfs in a Curtiss Wasp triplane rose 19,500 feet in 9 minutes, 42.4 seconds, what was his increase in elevation in feet per minute?



A CALIFORNIA FOOTHILL ORANGE GROVE

NUMBER OF PLANTS OR TREES FOR AN ACRE

DISTANCE APART	NO. OF PLANTS	DISTANCE APART	NO. OF PLANTS
4 × 4 inches	392,040	5 × 4 feet	2,178
6 × 6 inches	174,240	5 × 5 feet	1,742
9 × 9 inches	77,440	6 × 6 feet	1,210
1 × 1 feet	43,560	8 × 8 feet	680
1½ × 1½ feet	19,360	10 × 10 feet	435
2 × 2 feet	10,890	12 × 12 feet	302
2½ × 2½ feet	6,960	16 × 16 feet	170
3 × 3 feet	4,840	20 × 20 feet	108
4 × 1 feet	10,890	25 × 25 feet	69
4 × 2 feet	5,445	30 × 30 feet	48
4 × 3 feet	3,630	33 × 33 feet	40
4 × 4 feet	2,722	40 × 40 feet	27

ORAL DRILL

1. Read from the above table the number of plants to an acre if the plants are 9 inches apart.

2. How many to the acre if the plants are two and a half feet apart?

3. How many if four feet apart?

4. How many trees can be planted to the acre if they are set out 20 feet apart?

5. How many if planted 25 feet apart?

6. If lemon trees are planted 20 feet apart, how many will there be to the acre?

7. If orange trees are planted 22 feet apart, about how many will there be to the acre?

8. If walnut trees are planted 60 feet apart, about how many will there be to the acre?

RELATIVE WEIGHTS OF MATERIALS

MATERIAL	SPECIFIC GRAVITY	POUNDS PER CUBIC FOOT	POUNDS PER CUBIC INCH
Aluminum, Cast . . .	2.56	159.6	.0924
Aluminum, Sheet or Wire	2.67	166.5	.0963
Brass, Cast (75 % Cu.) .	8.4	523.8	.3031
Bronze (90 % Cu.) . . .	8.85	552	.3195
Copper, Sheet or Wire . .	8.93	556.8	.3222
Glass	2.62	163.4	.0945
Iron, Cast	7.22	450	.2604
Iron, Wrought	7.70	480	.2779
Lead, Cast	11.38	709.7	.4106
Mercury (60°)	13.58	846.8	.4900
Platinum	21.50	1342.1	.7767
Silver (Standard)	10.312	644	.3710
Steel (Bessemer)	7.854	489.6	.2834
Tin	7.29	455.1	.2634
Zinc	7.	436.5	.2536

ORAL DRILL

1. Read from the above table the weight of a cubic foot of cast iron.
2. Find the weight of a cubic foot of lead.
3. What does a cubic foot of cast brass weigh?
4. What is the weight of a cubic foot of aluminum?
5. What does a cubic foot of glass weigh?
6. Find from the table the weight of a cubic inch of cast iron.
7. A cubic inch of aluminum weighs how much?
8. How much does cast brass weigh per cubic inch?

LUMBER TABLE
Contents in Board Feet per Piece

SIZE IN INCHES	LENGTHS IN FEET							
	10	12	14	16	18	20	22	24
1 × 2	1 $\frac{2}{3}$	2	2 $\frac{1}{3}$	2 $\frac{2}{3}$	3	3 $\frac{1}{3}$	3 $\frac{2}{3}$	4
1 × 3	2 $\frac{1}{2}$	3	3 $\frac{1}{2}$	4	4 $\frac{1}{2}$	5	5 $\frac{1}{2}$	6
1 × 4	3 $\frac{1}{3}$	4	4 $\frac{2}{3}$	5 $\frac{1}{3}$	6	6 $\frac{2}{3}$	7 $\frac{1}{3}$	8
1 × 6	5	6	7	8	9	10	11	12
1 × 8	6 $\frac{2}{3}$	8	9 $\frac{1}{3}$	10 $\frac{2}{3}$	12	13 $\frac{1}{3}$	14 $\frac{2}{3}$	16
1 × 10	8 $\frac{1}{3}$	10	11 $\frac{2}{3}$	13 $\frac{1}{3}$	15	16 $\frac{2}{3}$	18 $\frac{1}{3}$	20
1 × 12	10	12	14	16	18	20	22	24
1 × 14	11 $\frac{2}{3}$	14	16 $\frac{2}{3}$	18 $\frac{2}{3}$	21	23 $\frac{1}{3}$	25 $\frac{2}{3}$	28
1 × 16	13 $\frac{1}{3}$	16	18 $\frac{2}{3}$	21 $\frac{1}{3}$	24	26 $\frac{1}{3}$	29 $\frac{1}{3}$	32
1 × 18	15	18	21	24	27	30	33	36
2 × 4	6 $\frac{2}{3}$	8	9 $\frac{1}{3}$	10 $\frac{2}{3}$	12	13 $\frac{1}{3}$	14 $\frac{2}{3}$	16
2 × 6	10	12	14	16	18	20	22	24
2 × 8	13 $\frac{1}{3}$	16	18 $\frac{2}{3}$	21 $\frac{1}{3}$	24	26 $\frac{2}{3}$	29 $\frac{1}{3}$	32
2 × 10	16 $\frac{2}{3}$	20	23 $\frac{1}{3}$	26 $\frac{2}{3}$	30	33 $\frac{1}{3}$	36 $\frac{2}{3}$	40
2 × 12	20	24	28	32	36	40	44	48
2 × 14	23 $\frac{1}{3}$	28	32 $\frac{2}{3}$	37 $\frac{1}{3}$	42	46 $\frac{2}{3}$	51 $\frac{1}{3}$	56
4 × 4	13 $\frac{1}{3}$	16	18 $\frac{2}{3}$	21 $\frac{1}{3}$	24	26 $\frac{2}{3}$	29 $\frac{1}{3}$	32
4 × 6	20	24	28	32	36	40	44	48
4 × 8	26 $\frac{2}{3}$	32	37 $\frac{1}{3}$	42 $\frac{2}{3}$	48	53 $\frac{1}{3}$	58 $\frac{2}{3}$	64
4 × 12	40	48	56	64	72	80	88	96
6 × 6	30	36	42	48	54	60	66	72
6 × 8	40	48	56	64	72	80	88	96
8 × 8	53 $\frac{1}{3}$	64	74 $\frac{2}{3}$	85 $\frac{1}{3}$	96	106 $\frac{2}{3}$	117 $\frac{1}{3}$	128
8 × 12	80	96	112	128	144	160	176	192
12 × 12	120	144	168	192	216	240	264	288

ORAL DRILL

1. Find from the above table the number of board feet in a board 1" × 8", 12 feet long.
2. How many board feet in a board 1" × 6", 18 feet long?
3. How many board feet in a board 1" × 18", 16 feet long?

4. How many board feet in a plank $2'' \times 12''$, 18 feet long?
5. How many board feet in a timber $12'' \times 12''$, 24 feet long?

Exercise

1. How much lumber in 10 pieces of $1'' \times 8''$, 12 feet long?
2. How much in 24 pieces of $4'' \times 4''$, 16 feet long?
3. How much in 50 pieces $8'' \times 8''$, 18 feet long?
4. How much in 40 pieces $4'' \times 8''$, 18 feet long?
5. How much in 52 pieces $4'' \times 4''$, 16 feet long, and 240 pieces $1'' \times 6''$, 16 feet long?
6. What will the lumber in number 5 cost at \$30 per M?

COMPOUND INTEREST TABLE

The Sum to Which One Dollar Will Amount

NO. OF YEARS	4 PER CENT	4½ PER CENT	5 PER CENT	5½ PER CENT	6 PER CENT	6½ PER CENT	7 PER CENT
1	1.040	1.045	1.050	1.055	1.060	1.065	1.070
2	1.082	1.092	1.103	1.113	1.124	1.134	1.145
3	1.125	1.141	1.158	1.174	1.191	1.208	1.225
4	1.170	1.193	1.216	1.239	1.262	1.286	1.311
5	1.217	1.246	1.276	1.307	1.338	1.370	1.403
6	1.265	1.302	1.340	1.379	1.419	1.459	1.501
7	1.316	1.361	1.407	1.455	1.504	1.554	1.606
8	1.369	1.422	1.477	1.535	1.594	1.655	1.718
9	1.423	1.486	1.551	1.619	1.689	1.763	1.838
10	1.480	1.553	1.629	1.708	1.791	1.877	1.967
11	1.539	1.623	1.710	1.802	1.898	1.999	2.105
12	1.601	1.696	1.796	1.901	2.012	2.129	2.252
13	1.665	1.772	1.886	2.006	2.133	2.267	2.410
14	1.732	1.852	1.980	2.116	2.261	2.415	2.579
15	1.801	1.935	2.079	2.232	2.397	2.572	2.759
16	1.873	2.022	2.183	2.355	2.540	2.739	2.952
17	1.948	2.113	2.292	2.485	2.693	2.917	3.159
18	2.026	2.208	2.407	2.621	2.854	3.107	3.380
19	2.107	2.308	2.527	2.766	3.026	3.309	3.617
20	2.191	2.412	2.653	2.918	3.207	3.524	3.870

ORAL DRILL

1. Find from the above table the sum to which \$1 will amount in ten years at 4 %.
2. To what will \$1 amount in seven years at 6 %?
3. To what will \$1 amount in five years at $5\frac{1}{2}$ %?
4. To what will \$10 amount in 12 years at 4 %?
5. To what will \$100 amount in 10 years at 6 %?
6. To what will \$10 amount in 20 years at $5\frac{1}{2}$ %?

POWERS, ROOTS, CIRCUMFERENCES AND AREAS OF CIRCLES

No.	SQUARE	CUBE	SQUARE ROOT	CUBE ROOT	No. = DIAMETER	
					Circum- ference	Area
1	1	1	1.0000	1.0000	3.142	0.7854
2	4	8	1.4142	1.2599	6.283	3.1416
3	9	27	1.7321	1.4422	9.425	7.0686
4	16	64	2.0000	1.5874	12.566	12.5664
5	25	125	2.2361	1.7100	15.708	19.6350
6	36	216	2.4495	1.8171	18.850	28.2743
7	49	343	2.6458	1.9129	21.991	38.4845
8	64	512	2.8284	2.0000	25.133	50.2655
9	81	729	3.0000	2.0801	28.274	63.6173
10	100	1000	3.1623	2.1544	31.416	78.5398
11	121	1331	3.3166	2.2240	34.558	95.0332
12	144	1728	3.4641	2.2894	37.699	113.0973
13	169	2197	3.6056	2.3513	40.841	132.7323
14	196	2744	3.7417	2.4101	43.982	153.9380
15	225	3375	3.8730	2.4662	47.124	176.7146
16	256	4096	4.0000	2.5198	50.265	201.0619
17	289	4913	4.1231	2.5713	53.407	226.9801
18	324	5832	4.2426	2.6207	56.549	254.4690
19	361	6859	4.3589	2.6684	59.690	283.5287
20	400	8000	4.4721	2.7144	62.832	314.1593

7. To what will \$10 amount in 10 years at 7 % ?
8. To what will \$100 amount in 7 years at 6 % ?
9. To what will \$1000 amount in 12 years at 7 % ?
10. To what will \$1000 amount in 20 years at $5\frac{1}{2}$ % ?

ORAL DRILL

1. Find from the table the square of 18.
2. Obtain from the table the square root of 12.
3. What is the cube root of 12 ?
4. What is the square of 16 ?
5. What is the square of 17 ?
6. What is the cube of 18 ?
7. What is the circumference of a circle whose diameter is 3 ?
8. What is the area of a circle whose diameter is 14 ?
9. What is the circumference of a circle whose diameter is 15 ?
10. What is the area of a circle whose diameter is 20 ?

CHAPTER VIII

SQUARE ROOT AND CUBE ROOT

Square Root

The **square root** of a number is one of the two equal numbers which multiplied together will produce the number. Thus, 7 is the square root of 49, and $\frac{1}{3}$ is the square root of $\frac{1}{9}$.

The square root of a number is usually indicated by writing the number under the *radical sign* $\sqrt{}$. The expression $\sqrt{81}$ is read "the square root of 81."

The square root of a fraction is found by taking the square root of the numerator and of the denominator. Thus, the square root of $\frac{9}{16}$ is $\frac{3}{4}$.

Since $1 = 1^2$, $100 = 10^2$, $10,000 = 100^2$, and so on, the square root of a number between 1 and 100 lies between 1 and 10; of a number between 100 and 10,000, lies between 10 and 100. In other words, the square root of a number expressed by *one* or *two* figures is a number of *one* figure; of a number expressed by *three* or *four* figures is a number of *two* figures; and so on. If therefore a number is divided into groups of two figures each, from right to left, the number of figures in its square root will be equal to the number of groups of figures. The last group to the left may consist of one figure or of two figures.

ORAL DRILL

What are the square roots of the following numbers?

1. 4	9. 25	17. 400	25. $\frac{9}{64}$
2. 16	10. 100	18. 441	26. $\frac{9}{25}$
3. 36	11. 169	19. 625	27. $\frac{25}{36}$
4. 144	12. 225	20. 10,000	28. $\frac{25}{81}$
5. 81	13. 196	21. $\frac{1}{4}$	29. $\frac{25}{64}$
6. 64	14. 256	22. $\frac{1}{16}$	30. $\frac{25}{49}$
7. 49	15. 324	23. $\frac{1}{36}$	31. $\frac{49}{64}$
8. 121	16. 289	24. $\frac{9}{16}$	32. $\frac{81}{100}$

EXAMPLE 1. Find the square root of 625.

$$\begin{array}{r}
 \begin{array}{r}
 20^2 = 400 \\
 2 \times 20 = 40 \\
 \underline{5} \\
 45
 \end{array}
 \begin{array}{r}
 \overline{6 \ 25} \bigg| 20 \\
 \underline{4 \ 00} \\
 2 \ 25 \quad 5 \\
 \underline{2 \ 25} \\
 25
 \end{array}
 \end{array}
 \quad \text{or, more briefly,}
 \quad
 \begin{array}{r}
 \overline{6 \ 25} \bigg| 25 \\
 \underline{4} \\
 45 \bigg| 2 \ 25 \\
 \underline{2 \ 25}
 \end{array}$$

Since 625 consists of two groups of figures, its square root is composed of two figures, tens and units. Since the square of tens is hundreds, 6 hundreds must be the square of at least 2 tens. Two

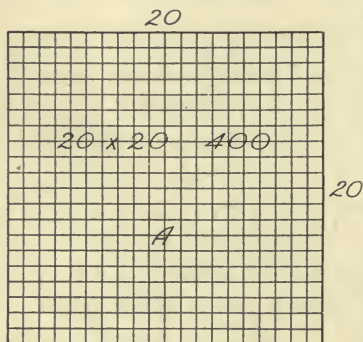
tens or 20 squared is 400, as

shown in figure A; and 625 — 400 leaves a remainder of 225.

The root 20, therefore, must be so increased as to use up the remainder and keep the figure a square.

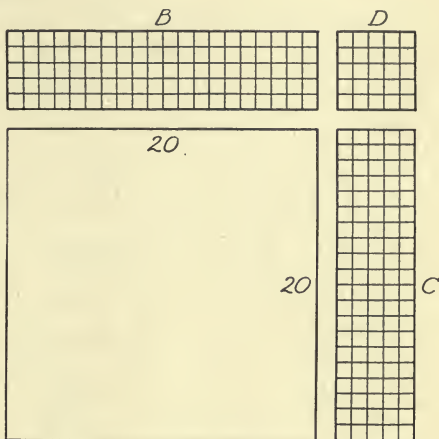
The necessary additions to A to keep it a square are the two rectangles B and C, and the small square D. B, C, and D contain 225 square units. Since the area

of D is small, if 225 is divided by 40, the combined length of B and



C , the quotient will indicate the approximate width of these additions. The quotient is 5; the entire length of B , C , and D is $20 + 20 + 5 = 45$ units; and the area of the additions is 5 times 45, or 225 square units. Since these three additions use up the remaining 225 square units and keep the figure a square, the side of the required square is 25 units, and the square root of 625 is 25.

In extracting square root arithmetically the following is a convenient rule.



Rule. — (1) Beginning at the units' place, separate the number into groups of two figures each.

(2) Find the largest square in the first group on the left and write its square root as the first figure in the answer.

(3) Subtract its square from the group and annex the second group to the remainder.

(4) For a trial divisor use twice the part of the root already found. Divide the remainder, omitting the last figure, by this trial divisor, and annex the quotient to the root and also to the trial divisor.

(5) Multiply the complete divisor by the second figure of the root and subtract the product from the remainder.

(6) Double the part of the root already found, for another trial divisor, and proceed as before.

(7) Continue this process until all of the groups have been used.

NOTE. — When a zero occurs in the root, annex a zero to the trial divisor, bring down another group, and proceed as before.

EXAMPLE 2. Find the square root of 501,264.

$$\begin{array}{r}
 313 \\
 \hline
 626
 \end{array}
 \quad
 \begin{array}{r}
 \widehat{50} \widehat{12} \widehat{64} | \underline{708} \\
 49 \\
 \hline
 1408 \quad \underline{11264} \\
 \underline{11264}
 \end{array}$$

Exercise

Find the square roots of the following numbers :

- | | | |
|---------|-------------|-------------|
| 1. 2209 | 6. 60,516 | 11. 467,856 |
| 2. 3136 | 7. 77,284 | 12. 558,009 |
| 3. 5184 | 8. 171,396 | 13. 683,929 |
| 4. 7056 | 9. 301,401 | 14. 767,376 |
| 5. 9025 | 10. 336,400 | 15. 819,025 |

Square Roots of Numbers Involving Decimals

Rule. — In extracting the square root of a number which is a decimal, or which is partly entire and partly decimal, or whose root involves a decimal, separate the number into groups of two figures, pointing off from the decimal point both to the left and to the right.

EXAMPLE 1. Extract the square root of .097969.

$$\begin{array}{r}
 \widehat{.09} \widehat{79} \widehat{69} | \underline{.313} \\
 9 \\
 \hline
 61 \quad 79 \\
 61 \\
 \hline
 623 \quad \underline{1869} \\
 \underline{1869}
 \end{array}$$

$$\begin{array}{r} 9 \overline{) 998.56} \overline{) 31.6} \\ \underline{9} \\ 98 \\ \underline{61} \\ 3756 \\ \underline{3756} \end{array}$$
$$\begin{array}{r}
 \overline{3.00} \overline{00} \overline{00} \overline{00} \overline{00} \mid \overline{1.73205} \\
 1 \\
 27 \quad \underline{200} \\
 343 \quad \underline{189} \\
 \quad \underline{1100} \\
 3462 \quad \underline{1029} \\
 \quad \underline{7100} \\
 \quad \underline{6924} \\
 34640 \quad \underline{1760000} \\
 \quad \underline{1732025} \\
 346405 \quad \underline{27975}
 \end{array}$$

1. 31.36	4. 54.76	7. 10	10. 2000	13. .060516
2. 37.21	5. 72.25	8. 27	11. 151.29	14. .128164
3. 42.25	6. 5	9. 1728	12. 1.8225	15. .169744

NOTE. — When the square root of a fraction is to be found, it is sometimes best to express the fraction as a decimal and extract the square root of the decimal.

EXAMPLE 4. Find the square root of $\frac{3}{5}$.

Expressed decimally, $\frac{3}{5}$ equals .6.

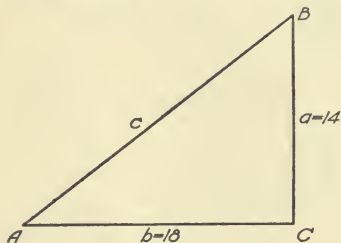
$$\begin{array}{r}
 \begin{array}{l} 147 \\ 1544 \\ 15486 \end{array} \left| \begin{array}{l} \widehat{.60} \widehat{00} \widehat{00} \widehat{00} \\ 49 \\ 1100 \\ 1029 \\ 7100 \\ 6176 \\ 92400 \end{array} \right. .7746 \text{ nearly}
 \end{array}$$

Exercise

Find the square roots of the following fractions :

- | | | | | |
|------------------|------------------|--------------------|---------------------|---------------------|
| 1. $\frac{1}{2}$ | 4. $\frac{3}{8}$ | 7. $\frac{7}{8}$ | 10. $\frac{17}{32}$ | 13. $\frac{11}{12}$ |
| 2. $\frac{1}{3}$ | 5. $\frac{2}{5}$ | 8. $\frac{5}{8}$ | 11. $\frac{4}{5}$ | 14. $\frac{21}{32}$ |
| 3. $\frac{3}{4}$ | 6. $\frac{5}{6}$ | 9. $\frac{11}{32}$ | 12. $\frac{5}{6}$ | 15. $\frac{27}{32}$ |

The finding of square root is necessary in solving such problems as the following :



EXAMPLE 1. The side a of a right triangle is 14 inches ; the side b is 18 inches. Find the length of the side ab .

From the Law of the Right Triangle, which states that

$$c^2 = a^2 + b^2$$

$$c = \sqrt{a^2 + b^2}$$

$$a^2 = 196$$

$$b^2 = 324$$

$$a^2 + b^2 = 520$$

$$c = \sqrt{520}$$

it follows that

Now

and

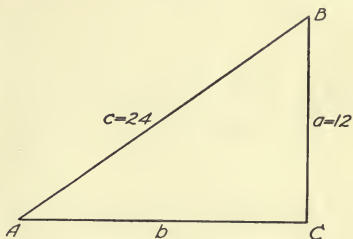
then

and

which is found to be 22.8035.

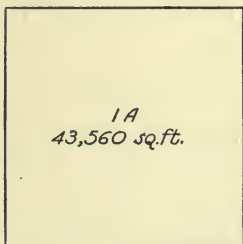
EXAMPLE 2. In a right triangle the hypotenuse c is 24 inches, and the side a is 12 inches. Find the length of the side b .

Now $b^2 = c^2 - a^2$,
 therefore $b = \sqrt{c^2 - a^2}$,
 $c^2 = 24^2 = 576$
 $a^2 = 12^2 = 144$
 then $c^2 - a^2 = 432$
 and $\sqrt{c^2 - a^2} = 20.7846$,
 the length of the side b .



EXAMPLE 3. Find the length of one side of a square field if the area is one acre.

If the area is 1 acre, or 43,560 square feet, the length of one side is $\sqrt{43560}$ feet.

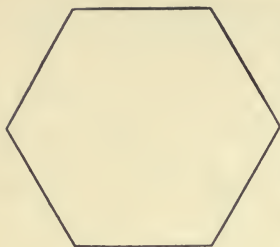


43560.0000	208.71
4	
408	3560
4167	3264
	29600
	29169
41741	43100
	41741

Exercise

1. If on a baseball diamond the distance between bases is 90 feet, how far is it from first base to third?

2. The height from the ground to a second-story window sill is 24 feet. If the foot of a ladder is set 8 feet from the house, how long must the ladder be to reach a foot beyond the sill?

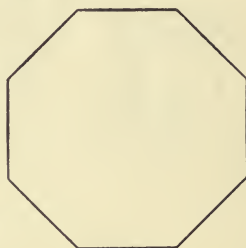


3. A table top in the form of a hexagon measures 20 inches on each side. What is the distance between parallel sides?

4. A table top is to be constructed in hexagonal shape. If it is to measure four feet between parallel sides of the hexagon, what must be the radius of the hexagon?

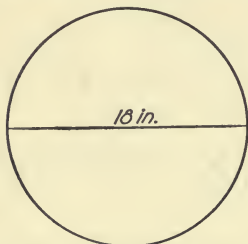
5. A table top is to be made of hexagonal shape and measure 6 feet between opposite edges. What will be the radius of the hexagon, and the length of each of its sides?

6. The distance between parallel sides of a table top octagonal in shape is 4 feet. What is the length of one of the sides of the octagon?



7. If each side of a regular octagon is 18 inches, what is the distance between any two of its parallel sides?

8. What are the dimensions, correct to the nearest inch, of the largest square timber that can be sawed from a log 18 inches in diameter?



9. If a circus-tent pole is 60 feet tall, and the guy rope from the top is attached to a stake 40 feet from its foot, what is the length of the guy rope?

10. If a straight roadway rises 220 feet in each mile of horizontal advance, what is the horizontal advance if the length of the road surface is 6000 feet?

11. What is the length in feet of one side of a square field if it contains 2 acres?

12. What is the length in feet of one side of a square field if it contains 10 acres?

13. What is the length of one side of a square field if its area is one half of a square mile?

14. A field is $\frac{1}{4}$ of a mile wide and $\frac{1}{2}$ of a mile long. How much less time will it take to cross it diagonally than to go the length of one side and an end if one travels at the rate of 12 miles an hour?

15. Two boys start from the same corner of a field half a mile square and race to the opposite corner. One travels along the outside of the field; the other crosses it diagonally. If the boy outside the field travels at the rate of 6 miles an hour, how fast must the other boy travel to beat him?

Cube Root of Arithmetical Numbers

Rule. — To extract the cube root of a number observe the following rule :

1. Separate the number into groups of three figures each, beginning at the decimal point and counting both toward the left and toward the right, completing the right-hand decimal group by annexing one or two ciphers if necessary.

2. By inspection or by trial find the greatest cube equal to or less than the number expressed by the left-hand group and write its cube root as the first figure of the answer.

3. From the left-hand group of the given number subtract the cube of the number just found, and bring down the next group as part of the remainder.

4. To three times the square of the number already found, annex two ciphers, thus forming the trial divisor.

5. Divide the above remainder by this trial divisor and write the quotient as the second figure in the answer.

6. To the trial divisor add three times the product of the number just obtained, multiplied by the part of the root previously found,

multiplied by 10; add also the square of the number just set down in the answer, thus forming the complete divisor.

7. Multiply the complete divisor by the number just set down in the answer and subtract the product from the above remainder; bring down the next group as part of the new remainder.

8. Repeat processes 4, 5, 6, and 7 until all of the groups in the given number have been used.

By annexing groups of three figures after the decimal point the process can be carried on indefinitely.

The number of groups of three figures each to the left of the decimal point indicates the number of figures the answer will have to the left of the decimal point. Thus it is seen that the cube root of 563,559.976 will have two figures to the left of the decimal point.

EXAMPLE 1. Find the cube root of 563,559.976.

			563559.976	82.6, the cube root
	5^3	512		
Trial divisor	19200		51559	
First addition	480			
Second addition	4			
Completed divisor	19684		39368	
Trial divisor	2017200		12191976	
First addition	14760			
Second addition	36			
Completed divisor	2031996		12191976	

EXAMPLE 2. Find the cube root of 0.157,464.

			0.157454	.54, the cube root
	5^3	125		
Trial divisor	7500		32454	
First addition	600			
Second addition	16			
Completed divisor	8116		32454	

Exercise

Find the cube roots of the following numbers :

- | | | | |
|-----------|------------|------------|-------------|
| 1. 512 | 6. 68.921 | 11. .343 | 16. .001728 |
| 2. 2197 | 7. 10.648 | 12. 409.6 | 17. .002744 |
| 3. 5832 | 8. 13.824 | 13. 156.25 | 18. .004713 |
| 4. 12,167 | 9. 21.952 | 14. 393.04 | 19. .006857 |
| 5. 35,937 | 10. 46.656 | 15. 911.25 | 20. .009261 |

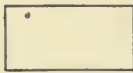
CHAPTER IX

MENSURATION

Plane Figures



SQUARE



RECTANGLE



PARALLELOGRAM



TRAPEZOID



RIGHT TRIANGLE



ISOSCELES TRIANGLE



EQUILATERAL TRIANGLE



IRREGULAR PENTAGON



REGULAR PENTAGON



REGULAR HEXAGON



REGULAR OCTAGON



PENTAGRAM



CIRCLE



CIRCLE
DIAMETER AND RADIUS



SEGMENT



SECTOR

Mensuration is the calculation of the areas, volumes, etc., of geometric figures when the dimensions of certain parts are given.

In the formulas of this chapter, small letters denote lengths ; capitals indicate areas or volumes. Thus, b stands for base, h for height, etc., while A represents area, S , surface, V , volume, etc.

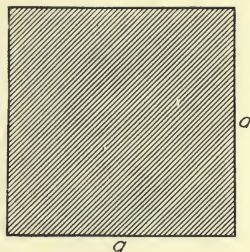
Squares

Rule.—To find the perimeter of a square, multiply the length of one side by 4.

$$p = 4a$$

Rule.—To find the area of a square, multiply the length of one side by itself.

$$A = a^2$$



EXAMPLE 1. Find the perimeter of a square each side of which is 9 feet.

$$\begin{aligned} p &= 4a \\ p &= 4 \times 9 = 36 \text{ feet} \end{aligned}$$

EXAMPLE 2. Find the area of a square each side of which is 8 feet.

$$\begin{aligned} A &= a^2 \\ A &= 8^2 = 64 \text{ square feet} \end{aligned}$$

EXAMPLE 3. Find the length of one side of a square whose area is 144 square feet.

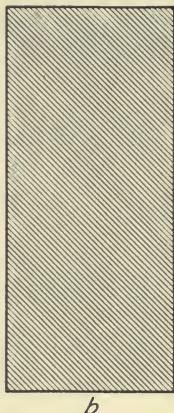
$$\begin{aligned} a^2 &= A \\ a &= \sqrt{A} \\ a &= \sqrt{144} = 12 \text{ feet} \end{aligned}$$

Exercise

1. Find the perimeter of a square if each side is 100 yards.
2. Find the area of a square if each side is 52 feet.
3. Find the area of a square if each side is 2.63 miles.
4. Find the length of one side of a square whose area is 19.6 square yards.

5. Find the length of one side of a square field whose area is 1 acre.
6. Find the area of a square field whose perimeter is 1 mile.
7. Which is the larger, a square field whose area is 2 acres, or a square field whose perimeter is 1200 feet?

Rectangles



Rule. — To find the perimeter of a rectangle, multiply the sum of the base and altitude by 2.

$$p = 2(a + b)$$

Rule. — To find the area of a rectangle, multiply the base by the altitude.

$$A = a \times b$$

Rule. — To find the length of the diagonal of a rectangle, extract the square root of the sum of the squares of the base and altitude.

$$d = \sqrt{a^2 + b^2}$$

EXAMPLE 1. Find the perimeter of a rectangle whose sides are 12 feet and 5 feet.

$$p = 2(a + b)$$

$$p = 2(12 + 5) = 34 \text{ feet}$$

EXAMPLE 2. Find the area of the rectangle of Ex. 1.

$$A = a \times b$$

$$A = 12 \times 5 = 60 \text{ square feet}$$

EXAMPLE 3. Find the length of a diagonal of the rectangle of Ex. 1.

$$d = \sqrt{a^2 + b^2}$$

$$d = \sqrt{144 + 25} = \sqrt{169} = 13 \text{ feet}$$

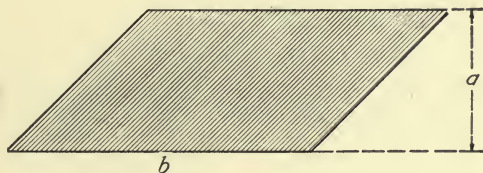
Exercise

1. Find the perimeter of a rectangle whose length is 18 feet and whose width is 15 feet.
2. Find the length of a rectangle whose width is $2\frac{1}{2}$ miles and whose area is 12 square miles.
3. Find the diagonal of a rectangle whose length and width are $5\frac{1}{2}$ miles and $3\frac{1}{2}$ miles respectively.
4. How many feet of picture molding are required to go entirely around a room 16 feet 4 inches long and 14 feet 8 inches wide?
5. Does a rectangle 220 feet long and 200 feet wide contain more than an acre?

Parallelograms

Rule. — To find the area of a parallelogram, multiply the base by the altitude.

$$A = a \times b$$

**Exercise**

1. Find the area of a parallelogram whose length is 18 feet and whose width is 13 feet.
2. Find the area of a parallelogram whose base and altitude are 14 yards and 10 yards respectively.
3. If the base of a parallelogram is $22\frac{1}{2}$ feet and its altitude is half as much, what is its area?

4. What must be the altitude of a parallelogram whose base is 100 yards and whose area is 1234 square yards?

5. If a parallelogram contains $2\frac{1}{2}$ acres and has a base 500 feet, what is its altitude?

6. In a parallelogram is it necessary that the base be greater than the altitude?

7. In a rectangle is it necessary that the base be greater than the altitude?

8. In a square is it necessary that the base be equal to the altitude?

9. If the area of a parallelogram is 222 square feet and its altitude is 22 feet, what is the length of its base?

10. Find the area of a parallelogram if its base is 15 yards and its altitude is 15 feet.

11. If the base of a parallelogram is 250 feet and its altitude is 150 feet, what is its area expressed as a decimal part of an acre?

12. If the base of a parallelogram is 360 feet and its area is 1 acre, what is its altitude?

Law of the Right Triangle

In a right triangle the square of the hypotenuse equals the sum of the squares of the other two sides.

$$c^2 = a^2 + b^2, \text{ or } c = \sqrt{a^2 + b^2}$$

$$a^2 = c^2 - b^2, \text{ or } a = \sqrt{c^2 - b^2}$$

$$b^2 = c^2 - a^2, \text{ or } b = \sqrt{c^2 - a^2}$$

The first statement above is sometimes called "the law of the right triangle."

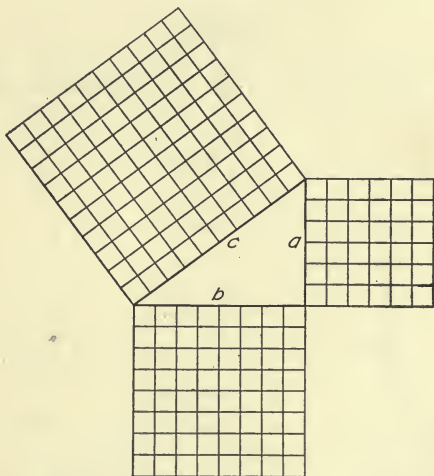
Rule. — To find the hypotenuse of a right triangle, extract the square root of the sum of the squares of the other two sides.

EXAMPLE 1. The two shorter sides of a right triangle are 8 feet and 14 feet. Find the length of the hypotenuse.

$$c^2 = a^2 + b^2$$

$$c^2 = 8^2 + 14^2 = 64 + 196 = 260$$

$$c = \sqrt{260} = 16.12 \text{ feet.}$$



EXAMPLE 2. The hypotenuse of a triangle is 25 feet. One side is 18 feet. What is the length of the other side?

$$a = \sqrt{c^2 - b^2}$$

$$= \sqrt{25^2 - 18^2} = \sqrt{625 - 324} = \sqrt{301} = 17.35 \text{ feet.}$$

Exercise

Given the sides of a right triangle as follows, find the length of the hypotenuse, correct to two decimal places:

- | | | |
|-------------------|---------------------|-----------------------|
| 1. 30 ft., 40 ft. | 3. 20 ft., 30 ft. | 5. 2 ft. 6 in., 3 ft. |
| 2. 45 ft., 60 ft. | 4. 1.5 in., 2.5 in. | 6. 3 ft. 8 in., 2 ft. |

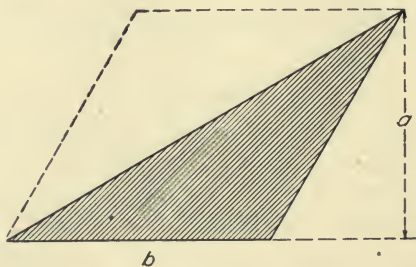
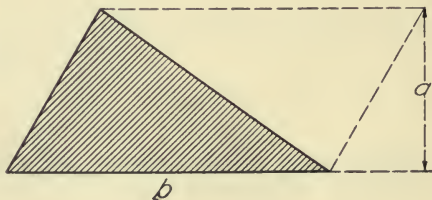
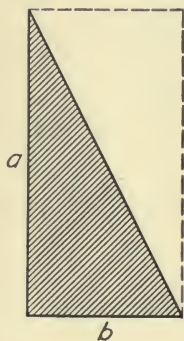
Given the hypotenuse and one side of a right triangle as follows, find the other side, correct to two decimal places :

7. 50 ft., 40 ft. 9. 10 ft., 6 ft. 11. 3 ft. 4 in., 2 ft.
8. 35 ft., 21 ft. 10. 12 in., 0.8 in. 12. 6 ft. 2 in., 5 ft.

Triangles

Rule. — To find the area of a triangle, multiply the base by $\frac{1}{2}$ the altitude.

$$A = \frac{a \times b}{2}.$$



EXAMPLE 1. Find the area of a triangle whose base is 12 yards and whose altitude is 8 yards.

$$A = \frac{a \times b}{2}$$

$$= \frac{12 \times 8}{2} = 48 \text{ square yards.}$$

EXAMPLE 2. Find the altitude of a triangle whose area is 36 square feet and whose base is 24 feet.

$$a = \frac{2 \times A}{b}$$

$$= \frac{2 \times 36}{24} = 3 \text{ feet.}$$

Exercise

Find the areas of triangles with the following bases and altitudes :

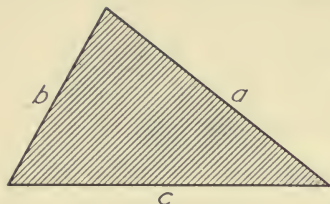
- | | | |
|--------------------|--------------------------------|-----------------------------|
| 1. 47 ft., 28 ft. | 4. $82\frac{1}{2}$ ft., 35 ft. | 7. 7 ft. 2 in., 3 ft. 1 in. |
| 2. 75 ft., 25 ft. | 5. 6 ft. 9 in., 27 in. | 8. 8 ft. 6 in., 4 ft. |
| 3. 160 rd., 25 rd. | 6. 3 yd. 2 ft., 4 ft. | 9. 12 ft., 3 ft. 3 in. |

Find the altitudes of triangles with the following areas and bases :

- | | |
|-------------------------|-------------------------|
| 10. 92 sq. ft., 23 ft. | 13. 500 sq. yd., 40 ft. |
| 11. 594 sq. ft., 9 rd. | 14. 5 acres, 45 rods |
| 12. 480 sq. ft., 48 ft. | 15. 2 sq. mi., 1000 ft. |

Rule. — To find the area of a triangle when the lengths of the three sides are given,

1. Add together the three sides and divide by 2.
2. From this half-sum subtract each side separately.



3. Multiply together the half-sum and these three remainders and extract the square root of the product.

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where a , b , and c are the three sides, and $s = \frac{a+b+c}{2}$.

EXAMPLE. Find the area of the triangle whose sides are 18, 24, 22.

$a = 18$	$s = 32$	$s = 32$	$s = 32$	32
$b = 24$	$a = 18$	$b = 24$	$c = 22$	14
$c = 22$	$s - a = 14$	$s - b = 8$	$s - c = 10$	128
$a + b + c = 64$				32
$s = 32$	$A = \sqrt{32 \times 14 \times 8 \times 10} = \sqrt{35,840}$			448
	$= 189.31$			80
				35840

Exercise

Find the areas of triangles with sides given as follows :

1. 20 ft., 30 ft., 40 ft.
2. 14 ft., 16 ft., 18 ft.
3. 18 ft., 22 ft., 26 ft.
4. 32 yd., 38 yd., 46 yd.
5. 52 yd., 62 yd., 72 yd.
6. 15 ft., 17 ft., 19 ft.
7. 25 yd., 35 yd., 45 yd.
8. 1.2 mi., 2.2 mi., 1.8 mi.
9. $\frac{1}{4}$ mi., $\frac{1}{8}$ mi., $\frac{1}{6}$ mi.
10. 32 meters, 38 meters, 44 meters

Trapezoids

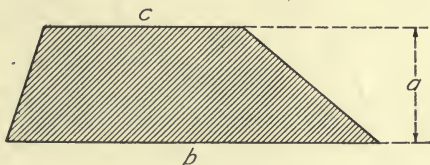
Rule. — To find the area of a trapezoid, multiply half the sum of the parallel sides by the altitude.

$$A = a \times \frac{b+c}{2}$$

EXAMPLE. Find the area of a trapezoid whose bases are 36 feet and 24 feet, and whose altitude is 18 feet.

$$A = a \times \frac{b + c}{2}$$

$$A = 18 \times \frac{36 + 24}{2} = 540 \text{ sq. ft.}$$



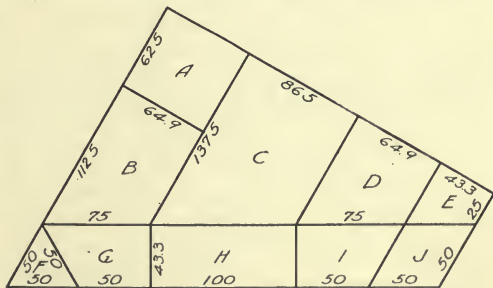
Exercise

Find the area of a trapezoid, having given :

1. Bases 37 ft. and 25 ft., altitude 19 ft.
2. Bases 1 ft. 5 in. and 1 ft. 11 in., altitude 4 ft. 8 in.
3. Bases $19\frac{1}{2}$ yd. and $26\frac{1}{2}$ yd., altitude 62 yd.
4. Bases 256 yd. and 144 yd., altitude 85 yd.
5. Bases 13.5 mi. and 7.7 mi., altitude 3.6 mi.

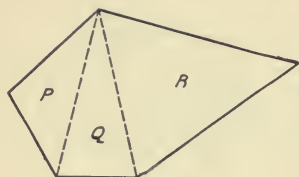
6. The bases of a trapezoid are 34 in. and 30 in. and the altitude is 20 in. Find the length of the side of a square having the same area.

7. The adjacent figure shows a tract of land subdivided into lots, the dimensions being given in feet. Find the area of each of



the lots in square feet. By how many square feet does the entire tract fall short of being an acre?

Irregular Polygons



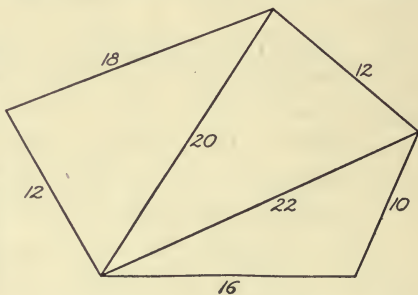
Rule. — To find the area of an irregular polygon, divide it into triangles and take the sum of their areas.

$$A = P + Q + R$$

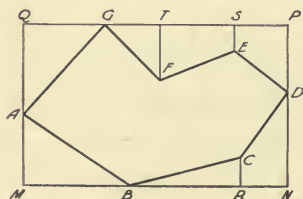
Exercise

1. The sides of a four-sided field $ABCD$, taken in the order AB , BC , CD , DA , are 25 rods, 60 rods, 52 rods, and 39 rods; the diagonal AC is 65 rods. How many acres are there in the field?

2. Find the area of a polygon whose sides are the lengths indicated in the accompanying figure. Are any of the triangles in the figure right-angled triangles? Are any of the triangles isosceles triangles? When it is difficult to enter a polygonal field for the purpose of measuring lines, the area can be found by inclosing the polygon in a rectangle, as shown below, and subtracting from the area of this rectangle the areas of the parts outside the polygon.



3. Find the area of the swamp $ABCDEFG$, having given:



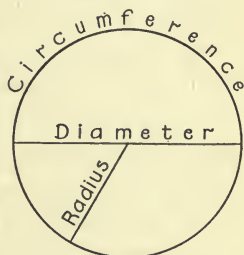
AM , 88 rd.	PS , 45 rd.
MB , 94 rd.	SE , 20 rd.
BR , 94 rd.	ST , 65 rd.
RC , 28 rd.	TF , 48 rd.
RN , 40 rd.	TG , 58 rd.
ND , 107 rd.	GQ , 60 rd.
DP , 104.5 rd.	QA , 123.5 rd.

Circles

π and Its Application. — The ratio of the length of the circumference of a circle to its diameter is always the same regardless of the size of the circle.

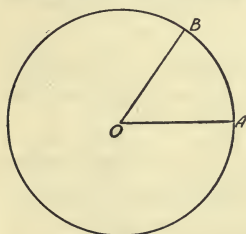
Measure as carefully as possible the circumference and the diameter of some such object as a smokestack, water main, dinner plate, milk pan, grindstone, or carriage wheel. Divide the first measurement by the second.

If the work has been carefully done the result will be about 3.14. By more careful methods an answer 3.1416 can be obtained.



The ratio of the circumference of a circle to its diameter is about $3\frac{1}{7}$ and is commonly designated by the Greek letter π .

The number $3\frac{1}{7}$ differs from the real or true value by about $\frac{1}{25}$ of one per cent and for most practical purposes is sufficiently close.



The **arc of a sector** bears the same ratio to the entire circumference that the angle of the sector bears to 360° .

Thus in the figure if the arc AB is $\frac{1}{6}$ of the circumference, the angle at O is $\frac{1}{6}$ of 360° , or 60° .

EXAMPLE. If a circular arc is 40 feet long and the radius of the circle is 50 feet, what is the angle at the center of the circle?

If the radius is 50 feet, the diameter is 100 feet.

$$\frac{\text{angle at the center}}{360^\circ} = \frac{\text{arc}}{\text{circumference}}$$

$$\frac{\text{angle at the center}}{360^\circ} = \frac{40}{100 \times 3\frac{1}{7}}$$

$$\text{angle at center} = \frac{360^\circ \times 40}{100 \times 3\frac{1}{7}} = 45.8 \text{ degrees}$$

Exercise

1. If a circular arc is 100 yards long and the angle at the center is 45° , what is the diameter of the circle?
2. If a circular arc is 4 feet long and the diameter of the circle is 12 feet, what is the angle at the center?
3. Assuming the radius of the earth to be 4000 miles, find the length of a degree of longitude; of a degree of latitude.
4. What fractional part of the circumference of a circle is intercepted by radii which meet at an angle of 30° ? of 45° ? of 60° ? of 90° ?
5. What decimal part of the area of a circle is the area of the square inscribed in the circle?
6. Through how many degrees of latitude does the state of California extend if its length from north to south is 750 miles?
7. Through how many degrees of longitude does California extend if its western extremity is 600 miles west of its eastern?
8. Through how many degrees of longitude does Alaska extend if it is 3000 miles between its eastern and western extremities?
9. If a foot racer runs 100 yards on a one-mile circular track, what angle at the center of the track will be made by joining it with the points where he starts and finishes?
10. How many degrees in the angle made by two radii which intercept an arc equal in length to the radius?

Where a greater degree of accuracy is desired, the following method is used.

Rule. — To find the circumference of a circle, multiply the diameter by π (about 3.1416).

$$c = 3.1416 d.$$

EXAMPLE 1. Find the circumference of a circle whose diameter is 15 ft.

$$c = (3.1416)(15) = 47.124 \text{ ft.}$$

EXAMPLE 2. Find the circumference of a circle whose radius is 4 ft.

$$d = 2r, \text{ then } d = 8 \text{ ft.}$$

$$c = (3.1416)(8) = 25.133 \text{ ft.}$$

Exercise

Find the circumferences of circles with diameters as follows:

- | | | | |
|-----------|-------------|-----------------------|-------------|
| 1. 9 in. | 4. 16 in. | 7. $2\frac{1}{2}$ yd. | 10. 2 mi. |
| 2. 12 in. | 5. 5.6 ft. | 8. $3\frac{1}{2}$ yd. | 11. .2 mi. |
| 3. 14 in. | 6. 10.5 ft. | 9. $5\frac{3}{4}$ yd. | 12. .02 mi. |

Find the circumferences of circles with radii as follows:

- | | | | |
|-----------|-------------|-------------|-------------|
| 13. 3 in. | 16. 3.5 ft. | 19. 2.3 yd. | 22. 3 mi. |
| 14. 5 in. | 17. 4.5 ft. | 20. 5.1 yd. | 23. .3 mi. |
| 15. 6 in. | 18. 7.2 ft. | 21. 7.3 yd. | 24. .03 mi. |

25. If the diameter of an automobile wheel is 36 inches, how many revolutions does it make in going 10 miles? (Use $\pi = \frac{22}{7}$.)

26. What is the diameter of a rope whose circumference is 3 inches?

27. If the diameter of the earth's equator is 7927 miles, what is its length?

28. What is the circumference of the circle that can be inscribed in a square 12 feet on a side?

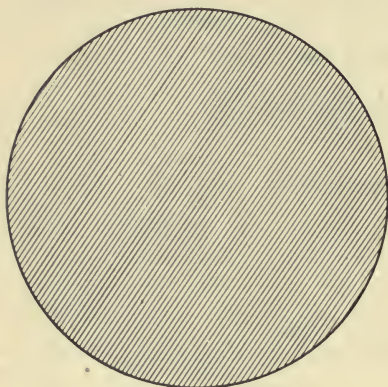
Area

Rule.— To find the area of a circle, multiply the square of the radius by 3.1416.

$$A = 3.1416 r^2.$$

Or, multiply the square of the diameter by .7854.

$$A = .7854 d^2.$$



EXAMPLE 1. Find the area of a circle whose radius is 4 inches.

$$\begin{aligned} A &= (3.1416)(16) \\ &= 50.265 \text{ sq. in.} \end{aligned}$$

EXAMPLE 2. Find the area of a circle whose diameter is 6 inches.

$$A = (.7854)(36) = 28.274 \text{ sq. in.}$$

EXAMPLE 3. Find the radius of a circle if its area is 2463 sq. ft.

$$A = 3.1416 r^2; \text{ then } r^2 = \frac{A}{3.1416} = \frac{2463}{3.1416}$$

and

$$r = \sqrt{\frac{2463}{3.1416}} = \sqrt{784} = 28 \text{ ft.}$$

Exercise

Find the areas of circles with radii as follows :

- | | | | |
|----------|------------|------------|-------------|
| 1. 3 in. | 4. 3.5 ft. | 7. 2.3 yd. | 10. 3 mi. |
| 2. 5 in. | 5. 4.5 ft. | 8. 5.1 yd. | 11. .3 mi. |
| 3. 6 in. | 6. 7.2 ft. | 9. 7.3 yd. | 12. .03 mi. |

Find the areas of circles with diameters as follows :

- | | | | |
|------------|--------------|------------------------|-------------|
| 13. 9 in. | 16. 16 in. | 19. $2\frac{1}{2}$ yd. | 22. 2 mi. |
| 14. 12 in. | 17. 5.6 ft. | 20. $3\frac{1}{2}$ yd. | 23. .2 mi. |
| 15. 14 in. | 18. 10.5 ft. | 21. $5\frac{3}{4}$ yd. | 24. .02 mi. |

25. What is the area of a cross section of a water pipe that is 3 inches in diameter?

26. What is the diameter of a circular plot of ground whose area is one acre?

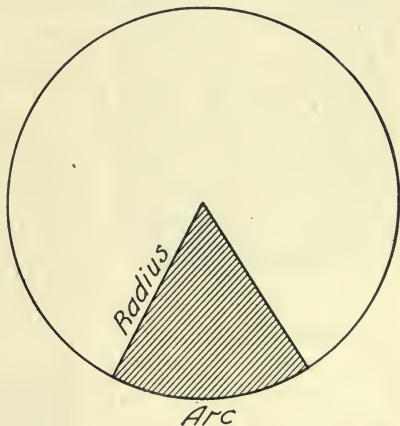
27. What is the radius of a circle whose area is one half an acre?

Sectors

Rule. — To find the area of a circular sector, multiply the radius by half the length of the arc.

$$A = \frac{a}{2} \times r.$$

EXAMPLE. Find the area of a sector whose radius is 6 ft. and whose angle is 14° .



Radius = 6 ft., then diameter = 12 ft.

Circumference of circle = $12 \times 3.1416 = 37.699$ ft.

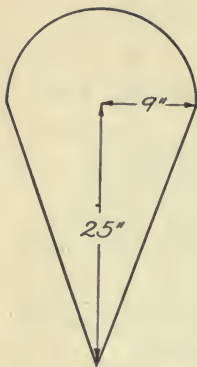
Arc of sector = $\frac{14}{360}$ of circumference

$$= \frac{14}{360} \times 37.699 = 1.47 \text{ ft.}$$

$$\text{Area of sector} = \frac{1}{2} \times 1.47 \times 6 = 4.41 \text{ sq. ft.}$$

Exercise

1. In a circle whose diameter is 3 ft., determine the area of a sector whose arc is 30° .



2. In a circle whose diameter is 10 ft., determine the area of a sector whose arc is 25° .

3. In a circle whose diameter is 14 ft., determine the area of a sector whose arc is 42° .

4. A kite is made of the shape shown in the diagram, the semicircle having a radius of 9 in., and the kite a total height of 34 in. Find the area of the kite.

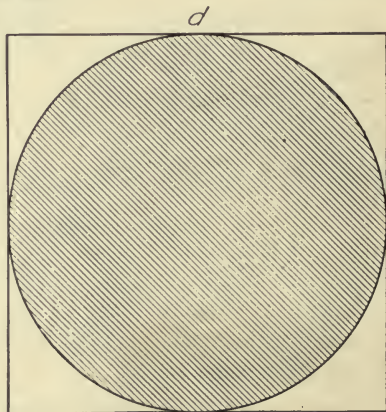
5. A 10-inch fan opens out into a sector whose angle is 130° . What is the area of the opened fan?

Ellipses

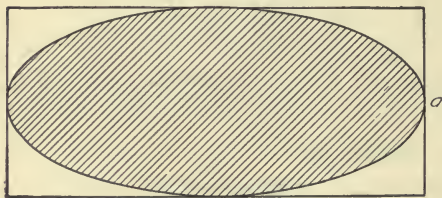
Rule. — To find the area of an ellipse, multiply the product of the diameters by .7854.

EXAMPLE. Find the area of an ellipse if the diameters are 14 in. and 8 in.

$$\begin{aligned} &.7854 \times 8 \times 14 \\ &= 87.965 \text{ sq. in.} \end{aligned}$$



CIRCLE



ELLIPSE

Exercise

Find the area of the ellipses whose diameters are the following:

1. 6 in., 8 in.
2. 10 in., 20 in.

- | | | |
|---------------------|---|---------------------|
| 3. 2.5 ft., 3.5 ft. | 5. 12 cm., 18 cm. | 7. .2 ft., .3 ft. |
| 4. 4.4 ft., 6.6 ft. | 6. $\frac{1}{2}$ mi., $\frac{3}{4}$ mi. | 8. .15 ft., .42 ft. |

Mensuration of Solid Figures

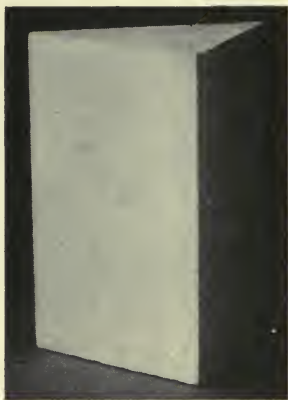
Volumes of Solid Figures. — The volumes of solid figures can be found by applying the following rules.

Rules. — 1. To find the volume of a prism or cylinder, multiply the area of the base by the altitude.

2. To find the volume of a pyramid or cone, multiply the area of the base by one third the altitude.

3. To find the volume of the frustum of a pyramid or cone, add to the sum of the areas of the bases the square root of their product, and multiply the sum by one third the altitude of the frustum.

4. To find the volume of a sphere, multiply the cube of the diameter by .5236.



TRIANGULAR PRISM

5. To find the volume of a spherical segment, add the square of its altitude to three times the square of the radius of the base of the segment; multiply this sum by the altitude, and the product by .5236.

6. To find the volume of a torus or cylindrical ring, multiply the area of the cross section by half the sum of the inner and outer circumferences.



FRUSTUM OF A CONE

Exercise

1. What is the volume of a cube 14 in. on an edge?
2. What is the volume of a block 3 ft. by 5 ft. by 6 ft.?



TRIANGULAR PYRAMID

3. A tank 2 ft. square on the bottom and 18 in. deep will hold how many gallons?

4. How many cubic yards of earth in a roadway cut 14 ft. wide, 42 ft. long, and 8 ft. 6 in. deep?

5. How many pounds of air in a room 24 ft. by 32 ft. by 14 ft. if 12 cu. ft. of air weigh 1 lb.?

Find the total areas and volumes of the following prisms:

6. Base 12 in. square, height 18 in.

7. Base $14\frac{1}{2}$ in. square, height $7\frac{1}{2}$ in.

8. Base a triangle with sides 5 in., 12 in., 13 in., height 26 in.

9. Base an equilateral triangle, sides 5 ft., height 22 ft.

10. Base a regular hexagon, sides 10 in., height 10 ft.

11. Find the weight of a cast-iron hexagonal prism of the dimensions given in the preceding example, if cast iron weighs 450 lb. per cubic foot.

12. Find the weight of a regular octagonal prism of granite, sides of the base being 6 in., and height 4 ft. 6 in., if granite weighs 170 lb. per cubic foot.



CIRCULAR CONE

13. Find the volume of a cylinder 10 in. in diameter and 12 in. high.

14. What is the weight of a cylindrical barrel of tar 2 ft. in diameter and 2 ft. 6 in. in height, if tar weighs 65 lb. per cubic foot?

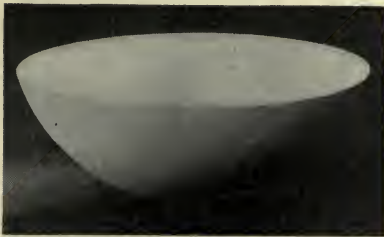
15. What is the weight of a granite pillar 2 ft. in diameter and 10 ft. high? (See Ex. 12.)

16. What is the capacity in gallons of a pipe-line 6 in. in diameter and 3 mi. long?

17. Find the weight of a mile of aluminum wire $\frac{1}{4}$ in. in diameter, if aluminum weighs 165 lb. per cubic foot.

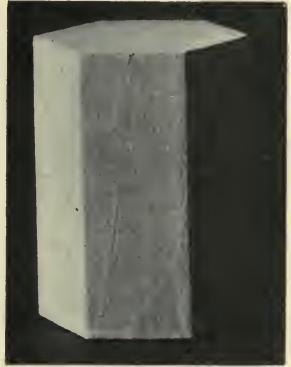
18. A hollow cast iron cylinder open at both ends is 6 ft. long and 4 ft. in diameter, outside measurements. What will it weigh if it is 1 in. thick? (See Ex. 11.)

19. How many pyramids of ice cream can be made from 5 gallons of material if the pyramids have square bases $2\frac{1}{2}$ in. on a side, and an altitude of $1\frac{1}{2}$ in.?



SPHERICAL SEGMENT

21. If the conical crater of a volcano is $2\frac{1}{2}$ mi. in diameter and $\frac{1}{4}$ mi. deep, how many tenths of a cubic mile of material will it take to fill it?



HEXAGONAL PRISM

20. A conical shaped heap of grain is 4 ft. high and the diameter of the circle covered by its base is 10 ft. If the grain is put into a bin 4 ft. square, how deep will it be and how many bushels are there?

22. A quantity of fine coal is thrown up against the side of a bin. The highest point of the coal is 6 ft., where it touches



HOLLOW CYLINDER

the side of the bin, and the slant height of the heap is 9 ft. If the base of the heap is semi-circular in shape, how many tons does the pile contain if a ton of coal occupies 36 cu. ft.?

23. Find the volume of a pyramid whose base and faces are all equilateral triangles with sides 10 in. long.

24. Find the volume of the frustum of a square pyramid if the sides of its bases are 40 ft. and 16 ft., and the altitude of a lateral face is 20 ft.

25. A bucket is 16 in. deep, 18 in. in diameter at the top, and 12 in. in diameter at the bottom. How many gallons will it hold?

26. Find the weight of a pine flagstaff 120 ft. long, 24 in. in diameter at the base and 8 in. at the top, if pine weighs 35 lb. per cubic foot.

27. Determine the capacity in gallons of a pail 14 in. in diameter at the top, 10 in. in diameter at the bottom, 10 in. high. What is the capacity of a pail similar in shape but with all the dimensions half as great?



QUADRANGULAR PRISM

28. How many cubic yards of earth will be required to fill 10,000 flower-pots the inside diameter of whose upper

and lower bases are 10 in. and 6 in. and whose depth is 10 in. ?

29. Find the volume of a sphere 14 in. in diameter.

30. What is the weight of a cast brass sphere 6 in. in diameter if a cubic foot of brass weighs 500 lb. ?

31. What is the weight of an ivory sphere 3 in. in diameter, if ivory weighs 114 lb. per cubic foot ?

32. Find the weight of a cast-iron dumb-bell if the bells are 5-inch spheres connected by a cylindrical bar $5\frac{1}{4}$ in. long and $1\frac{3}{8}$ in. in diameter. (See Ex. 11.)

33. What is the weight of 10,000 marbles $\frac{3}{4}$ in. in diameter if the material of which the marbles are made weighs 170 lb. per cubic foot ?

34. At the rate of 250 cu. ft. per minute how long will it take to inflate a spherical balloon 51 ft. in diameter ?



TORUS OR CYLINDRICAL RING

CHAPTER X

GRAPHICAL REPRESENTATION

It frequently happens that an idea can be expressed very much more clearly and briefly by a drawing than by words. Thus, the idea of a square is conveyed instantly by a picture of a square $\square$. The drawing or diagram is called a *graphical representation*.

Many illustrations of this sort are doubtless already familiar to the student. Tally marks, ~~||||~~ ~~||||~~, the indication of the hours on the clock-face by the numerals, I, II, III, IIII, etc., the determination of the pitch of a note by its location on the musical staff, , the crescendo sign < , diminuendo > , etc., are all graphic in character, as also are the indication of direction by an arrow $\rightarrow$, the north and south line $\nwarrow \nearrow$, conventional signs used in map drawing, like ~~++++~~ for railroad, ~~|||||~~ for grass, astronomical symbols such as $\oplus$ for earth, $\text{\textcircled{C}}$ for moon, the symbols of geometry, $\triangle$ for triangle, $\odot$ for circle, and the like.

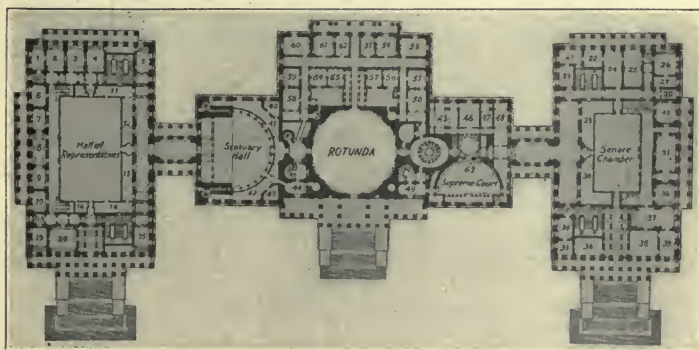
A geographical map is an example of line representation wherein distances are plotted to some convenient scale. In every case the scale used will depend upon the amount of territory to be shown and on the desired size of the map itself.

Line representation is the basis of drawing and of all mechanical work. The floor plan of a house, for example,



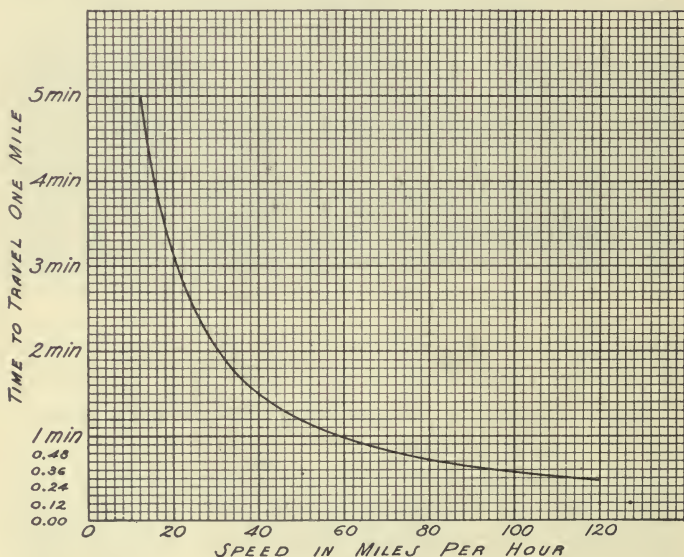
NATIONAL CAPITOL AT WASHINGTON, D. C.

and its elevation, are graphical representations of the location of walls, doorways, windows, etc. The accompanying figures show a general view of the National Capitol together with its floor plan.



FLOOR PLAN OF THE NATIONAL CAPITOL

Such data as valuation of exports and imports, increase in population, relative lengths of rivers, heights of mountains, changes in temperature, amounts of rainfall, variations in the price of foodstuffs, and the like, can be presented much more effectively by means of diagrams than by columns of figures.



A relation like that which exists between the speed of a vehicle and the time spent in traveling one mile is best shown diagrammatically, as in the accompanying figure.

Graphic diagrams are of a great variety of forms, and in determining upon the scheme of representation to be used in a particular case, the student should select that type which will set forth in the most telling fashion the essential facts of the data to be presented.

Exercise

1. Using the figures in the table below, prepare a graphical diagram showing the relative population of the world's largest cities.

POPULATION OF THE WORLD'S LARGEST CITIES

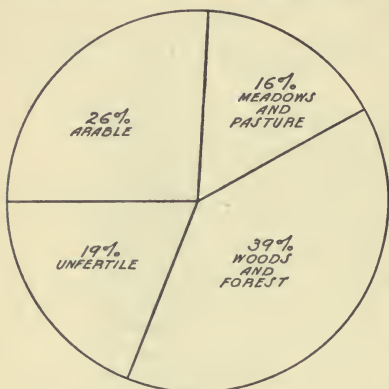
CITY	POPULATION
London	7,540,000
New York	4,767,000
Paris	3,850,000
Berlin	3,430,000
Chicago	2,185,000
Vienna	2,030,000
Petrograd	1,870,000
Tokio	1,820,000
Philadelphia	1,549,000
Moscow	1,468,000
Buenos Aires	1,314,000
Calcutta*	1,216,000
Constantinople	1,200,000

2. Using the figures in the table below, prepare a diagram showing the population of the United States at each census.

POPULATION OF THE UNITED STATES AT EACH CENSUS

CENSUS YEAR	POPULATION
1790	3,929,214
1800	5,308,483
1810	7,239,881
1820	9,638,453
1830	12,866,020
1840	17,069,453
1850	23,191,876
1860	31,443,321
1870	38,558,371
1880	50,155,783
1890	62,947,714
1900	75,994,575
1910	91,972,266

Sometimes a circular area divided into sectors offers itself as the best means of setting forth data, especially if the relation is a simple one. Illustrations of this type are shown in the accompanying diagrams. The first sets forth the facts of ground utilization in European Russia; the second shows to what extent the annual cut exceeds the annual forest growth in the United States.

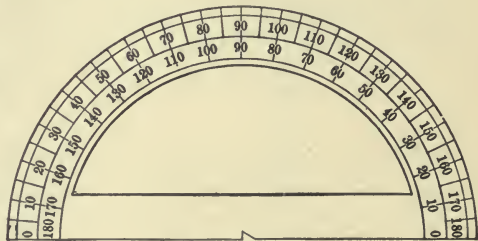


GROUND UTILIZATION IN EUROPEAN
RUSSIA

For laying out the angles at the centers of the circles in such diagrams as these, an instru-

ment called a *protractor* is sometimes used. By joining the notch 0 of the protractor to each graduation mark, a set of angles is obtained at 0 of one degree each.

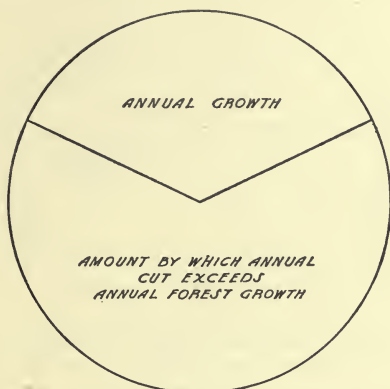
To measure a given angle, place the notch of the protractor at the vertex of the angle, and the base line along one side of the angle. The other side of the angle then indicates on the protractor the number of degrees in the angle.



A PROTRACTOR

To draw an angle of a given number of degrees, place the base of the protractor along a straight line and mark on

the line the position of the notch 0. Then place the pencil at the required graduation mark and (after removing the protractor) join the point so marked to 0.



THE EXTENT TO WHICH THE ANNUAL CUT EXCEEDS
THE ANNUAL FOREST GROWTH IN
THE UNITED STATES

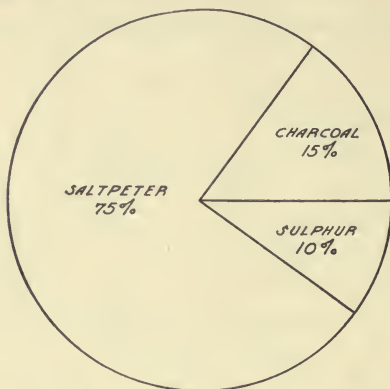
EXAMPLE 1. Show graphically by the sector method the relative amounts of the various constituents of black gunpowder if it is made up of

Sulphur	10 per cent
Charcoal	15 per cent
Salt peter	75 per cent

The circumference of a circle contains 360 degrees. The parts into which it is to be divided in this case are 10 hundredths of it for sulphur, 15 hundredths of it for charcoal, and 75 hundredths of it for salt peter. That is,

Sulphur	$.10 \times 360^\circ$, or 36°
Charcoal	$.15 \times 360^\circ$, or 54°
Salt peter	$.75 \times 360^\circ$, or 270°

If a circle is drawn and these angles laid off, the accompanying figure will result.



RELATIVE AMOUNTS OF THE VARIOUS CON-
STITUENTS OF BLACK GUNPOWDER

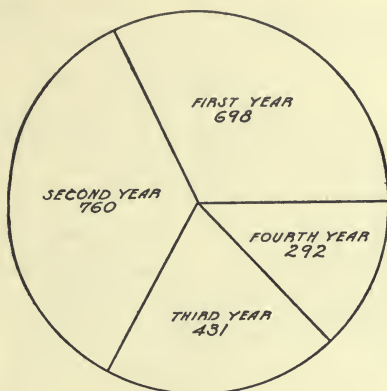
EXAMPLE 2. Show in similar manner the relative numbers of students in the four classes of the Los Angeles Polytechnic High School if the numbers are as follows :

First-year students	698
Second-year students	760
Third-year students	431
Fourth-year students	292

It is frequently an aid in making the drawing if the figures to be used are tabulated in some such way as the following :

YEAR	NO. ENROLLED	PERCENTAGE OF TOTAL	NO. OF DEGREES IN SECTOR
First	698	32	112
Second	760	35	126
Third	431	20	72
Fourth	292	13	47
Total	2181	100	360

If the angles in the fourth column are laid off on a circumference of convenient size, the figure will have the following appearance.



In the construction of a graphic chart, care should be exercised in the selection of the lengths of the units to be used so that the figure or diagram may not be of inconvenient size or shape.

Exercise

1. Observing the precaution just given, prepare a diagram showing the temperatures at which various substances melt.

TEMPERATURES AT WHICH VARIOUS SUBSTANCES MELT

SUBSTANCE	MELTING POINT DEGREES CENTIGRADE
Ice	0
Sulphur	114
Tin	233
Lead	330
Zinc	433
Aluminum	650
Silver	954

SUBSTANCE	MELTING POINT DEGREES CENTIGRADE
Gold	1063
Copper	1100
Cast iron	1200
Platinum	1775
Iridium	1950

2. Prepare a graph setting forth the following facts :

ABUNDANCE OF THE ELEMENTS IN NATURE

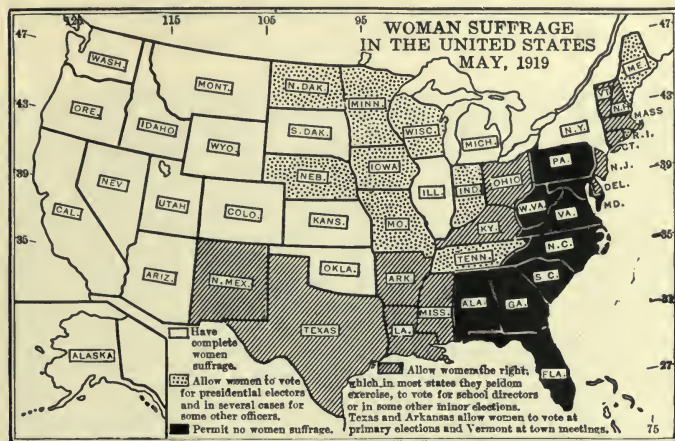
LIST OF ELEMENTS IN ORDER OF ABUNDANCE	COMPOSITION OF THE EARTH'S CRUST INCLUDING THE OCEAN AND THE ATMOSPHERE
Name	Per Cent
Oxygen	49.98
Silicon	25.30
Aluminum	7.26
Iron	5.08
Calcium	3.51
Magnesium	2.50
Sodium	2.28
Potassium	2.23
All others	1.86

Frequently the pictorial idea can be effectively used, as illustrated below.



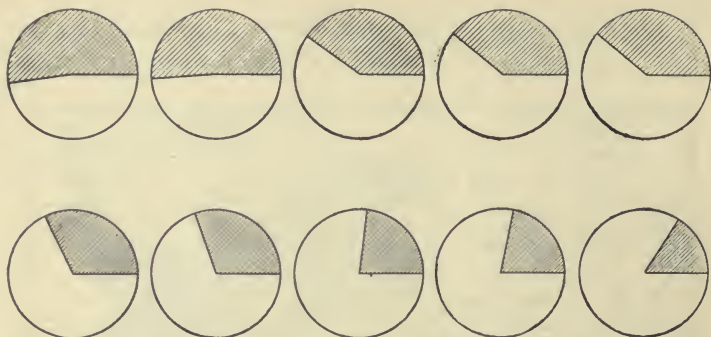
EGG YIELDS.— A record yearly yield (314 eggs) compared with the yield of an average hen (135 eggs)

The character of the vote at an election can be set forth by such an arrangement as that shown in the accompanying map.



In the diagram setting forth the facts of negro population, observe that the shading of the portion of the circle representing the negro population is itself an aid to the easy interpretation of the diagram.

STATES	NEGRO POPULATION	TOTAL POPULATION	PER CENT
So. Carolina	782,000	1,515,000	.52
Mississippi	908,000	1,797,000	.51
Georgia	1,035,000	2,609,000	.40
Alabama	827,000	2,138,000	.39
Louisiana	651,000	1,656,000	.39
Virginia	661,000	2,062,000	.32
No. Carolina	624,000	2,206,000	.28
Arkansas	367,000	1,574,000	.23
Tennessee	480,000	2,185,000	.22
Texas	621,000	3,897,000	.16



RELATIVE WHITE AND COLORED POPULATION IN TEN OF THE SOUTHERN STATES.— First line: South Carolina, Mississippi, Georgia, Alabama, Louisiana. Second line: Virginia, North Carolina, Arkansas, Tennessee, Texas

Exercise

From data probably obtainable from the local Chamber of Commerce, prepare graphical diagrams setting forth concerning the city in which you live, information along the following lines:

1. Variation in temperature during twenty-four hours.
2. Monthly rainfall for a period of years.
3. The hourly electric output at some near-by power house.
4. Daily consumption of gas.
5. Daily consumption of meat, milk, eggs, or bread.

Exercise

Prepare graphical charts, obtaining the data where you can, setting forth the following:

1. The number of passengers who travel on the street cars at various hours during the day, in your city.
2. Bank clearings in your city by years, months, or days for the past ten years.

3. Post-office business in the largest three cities in your state.
4. Values of various agricultural productions of your county.
5. Values of various mineral productions of your state.



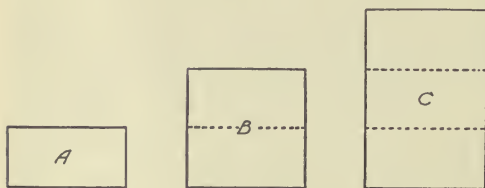
LIGHTNING FLASH. — A natural graph

Abundance of material suitable for graphic representation can be found in such publications as the *Statistician's Year Book*, *World Almanac*, *Tribune Almanac*, *Scientific American Reference Book*, and any geographical statistical atlas. Much material of this kind can usually be obtained from the local Chamber of Commerce, Bureau of Mines, Development Board, the newspaper offices, and the banks. The student will find numerous other types of graphical representation in various magazines, especially those devoted to engineering and construction work. Indeed, scarcely a book or technical article is written these days wherein the author does not have recourse to pictorial charts or diagrams.

CHAPTER XI

RATIO AND PROPORTION

Ratio. — In the accompanying figure it is apparent that A is one half as large as B , and one third as large as C .



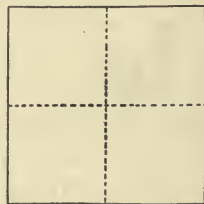
It is customary to say, then, that the ratio of A to B is one half, and that the ratio of A to C is one third. Similarly

it is evident that B is two thirds as large as C . That is, the ratio of B to C is two thirds.

Ratio therefore has to do with the relative size of things and is the numerical relation between two quantities of the same kind.

Thus, since 6 is half as great as 12, the ratio of 6 to 12 is one half. The ratio of 2 pounds to 3 pounds is two thirds; the ratio of one foot to one yard is one third.

A ratio can be expressed in a variety of ways. Thus, if one number is half as great as another, their ratio can be expressed *graphically*



thus ; or if a certain quantity is one fourth as great as another, their ratio can be expressed as in the accompanying figures.

Ratio can be expressed *numerically* in any one of several ways. For example, the ratio of 2 to 3 can be expressed as $\frac{2}{3}$, $2/3$, $2 \div 3$, or $2:3$. Such an expression as $2:3$ is read "the ratio of 2 to 3," or "2 is to 3."

Definitions. — In the ratio $2:3$, 2 and 3 are called the **terms** of the ratio. Expressed in fractional form the same ratio reads $\frac{2}{3}$ or $2/3$. The numerator or first term of the ratio is called the **antecedent**. The denominator or second term of the ratio is called the **consequent**. Thus, in the ratio $4:5$, 4 is the antecedent and 5 is the consequent.

ORAL DRILL

In each of the following examples determine the ratio of the first-named quantity to the second:

- | | |
|---|-----------------------------------|
| 1. 3 ft. to 7 ft. | 12. \$ 3.50 to \$ 14.00 |
| 2. 4 yd. to 9 yd. | 13. $14\frac{1}{2}$ lb. to 29 lb. |
| 3. 2 lb. to 14 lb. | 14. 29 lb. to $14\frac{1}{2}$ lb. |
| 4. 2 ft. to 5 yd. | 15. 1 oz. to 1 lb. (Avoir.) |
| 5. 5 ft. to 1 yd. | 16. 1 oz. to 1 lb. (Troy) |
| 6. 10 lb. to 6 lb. | 17. 1 sq. ft. to 1 sq. yd. |
| 7. $\frac{1}{2}$ mi. to 2 mi. | 18. 1 sq. yd. to 1 sq. ft. |
| 8. 14 in. to 3 yd. | 19. $4\frac{1}{2}$ tons to 75 lb. |
| 9. 10 yd. to 3 in. | 20. 1 kilometer to 1 mi. |
| 10. \$ 7.50 to \$ 3.75 | 21. 1 decimeter to 1 in. |
| 11. \$ 14.00 to \$.50 | 22. 1 millimeter to 1 in. |
| 23. The area of a square 1 ft. on a side to the area of a square 2 ft. on a side. | |
| 24. The area of a square 2 ft. on a side to the area of a square 1 ft. on a side. | |
| 25. The volume of a 1-ft. cube to the volume of a 2-ft. cube. | |

26. The volume of a 12-in. cube to the volume of a 4-in. cube.
27. The perimeter of a square to one side.
28. The perimeter of an equilateral triangle to one side.
29. The diagonal of a square to one side.
30. The side of a square to the diagonal.
31. The diameter of a circle to its radius.
32. The radius of a circle to its diameter.
33. The circumference of a circle to its diameter.
34. The diameter of a circle to its circumference.
35. The circumference of a circle to its radius.
36. The area of a circle to the area of a square whose side is the same as the diameter of the circle.

Rates. — In a ratio the things compared are of the same kind. It frequently happens, however, that things of different kinds must be compared, and this comparison gives rise to the idea of a **rate**. Such expressions as so many threads to the inch, so many revolutions per minute, etc., are of common occurrence and are called *rates*.

Exercise

1. If a wheel makes 2000 revolutions per minute, how many does it make in 10 seconds?
2. If a vehicle travels 55 miles an hour, what is the distance traveled in 15 minutes? in 15 seconds?
3. If there are 8 threads to the inch in a nut, how far will it advance in making 20 turns?
4. If telephone poles are placed 100 feet apart, how many poles are there to the mile?
5. If the price of gas is \$.65 per M, what is the charge for 2260 cubic feet of gas?

6. If lumber is worth \$22.50 per M, what are 650 feet worth?

7. If a pump delivers 500 gallons per minute, what is the delivery per second? per hour?

8. If an automobile will run 25 miles on a gallon of gasoline, what is the amount of gasoline consumed on a 1000-mile trip?

9. If trees are planted 64 to the acre, how many trees are there in 40 acres? in a square mile?

10. If there are 3 quarter notes or their equivalent, per measure, how many quarter notes or equivalent are there in a piece of music of 32 bars?

11. If the yield of potatoes is 400 bushels to the acre, what is the yield from $2\frac{1}{2}$ acres? from $\frac{1}{2}$ acre?

12. If a certain milk sample yields $2\frac{1}{2}$ pounds of butter fat per gallon, how many pounds of butter fat will 100 gallons of such milk yield?

13. If a stenographer writes at the rate of 225 words per minute, how long at the same rate would it take her to write 2000 words?

14. If a spindle tapers one sixteenth of an inch to the inch, what is the amount of taper per foot?

15. If a mechanically operated hack-saw makes 90 strokes per minute, what is the number per hour?

16. If ammunition is served at the rate of 40 rounds per man, what is the number of rounds supplied to 250 men? How many men will 10,000 rounds serve?

17. At \$3.25 per day, how many days' work will \$100 pay for?

18. If a change of elevation of about 295 meters makes a difference of 1° Centigrade in the boiling point, what is the boiling point at a place whose elevation is 2 miles? $1\frac{1}{2}$ miles?

19. If the fall in the barometer is .1 inch for every 100 feet of elevation, what is the fall due to an increase in elevation of 2600 feet? of 6200 feet?

20. If the yield of alfalfa is one and one-half tons per acre for each cutting, and the crop is cut seven times each year, what is the yield per year on 12 acres?



HARVESTING ALFALFA

A knowledge of ratio is essential to a proper understanding of the mathematics of all industrial and mechanical pursuits. For example, mechanical advantage is defined in terms of ratio; specific gravity is defined in terms of ratio, and such expressions as “gear ratio” and “coefficient of expansion” are of frequent occurrence.

The term **specific gravity** is used to denote the ratio between the weight of a body and the weight of an equal volume of water. If a cubic foot of lead weighs 11.4

times as much as a cubic foot of water, its specific gravity is said to be 11.4. Thus, 11.4 is the number that expresses the *ratio* which the weight of a given volume of lead bears to the weight of an equal volume of water.

EXAMPLE 1. If a cubic foot of water weighs 62.3 pounds and the specific gravity of brass is 8.6, what is the weight of a cubic foot of the latter?

$$\begin{array}{rcl}
 62.3 & \text{Weight of 1 cubic foot of water} & \\
 \underline{8.6} & \text{Specific gravity of brass} & \\
 373.8 & & \\
 4984 & & \\
 \hline
 535.78 & 536 \text{ pounds, nearly} &
 \end{array}$$

EXAMPLE 2. What is the weight of a cubic foot of cork if the specific gravity of cork is 0.24?

$$\begin{array}{rcl}
 62.3 & \text{Weight of 1 cubic foot of water} & \\
 \underline{.24} & \text{Specific gravity of cork} & \\
 2492 & & \\
 1246 & & \\
 \hline
 14.952 & 15 \text{ pounds, nearly} &
 \end{array}$$

Exercise

1. If the specific gravity of wrought iron is 7.8 and water weighs 62.3 pounds per cubic foot, find the weight of a cubic foot of wrought iron.

2. If the specific gravity of lead is 11.4, what is the weight of 5 cubic feet of it?

3. If platinum has a specific gravity of 21.4, find the weight of a cubic inch of it.

4. If earth has a specific gravity of 1.6, what is its weight per load (cubic yard)?

5. If the specific gravity of wet sand is 2, what weight of it will a box 4 feet wide, 1 foot deep, and 8 feet long hold?

6. If concrete has a specific gravity of 2.5, what is the weight of concrete per cubic yard?

7. If the weight of a sphere is a little more than half as much as a cube of the same dimensions, find the weight of 10 lignum vitae nine-pin balls 6 inches in diameter. (Specific gravity lignum vitae 1.3.)

8. If the specific gravity of ice is .9, what is the weight of a block of it 4 feet wide, 4 feet thick, and 8 feet long?

9. Olive oil has a specific gravity of about .9. What is the weight of 5 gallons of it?

10. What is the weight of 1000 gallons of petroleum if its specific gravity is .8?

Proportion

A **proportion** is a statement of equality between two equal ratios. Thus $\frac{3}{4} = \frac{6}{8}$ is a proportion. It is equivalent to saying that the ratio of 3 to 4 is equal to the ratio of 6 to 8.

The numbers 3, 4, 6, and 8 are called the **terms** of the proportion. When two ratios are equal the four terms are said to be *in proportion*.

The numbers 4 and 6 are called the **means**; the numbers 3 and 8 are called the **extremes**.

The proportion $\frac{3}{4} = \frac{6}{8}$ can also be written in the form $3:4=6:8$. It is read: "3 is to 4 as 6 is to 8."

In the above proportion it is obvious that 3×8 equals 4×6 . In the proportion $2:5=4:10$ it is also obvious that 2×10 equals 5×4 . This relation will be observed to hold true in every proportion.

Rule.—In every proportion, the product of the extremes is equal to the product of the means.

The word *proportion* is sometimes used as synonymous with ratio, as for example in the expressions, "the proportion of copper to zinc in brass," "the proportion of business men who succeed," etc.

The process of finding from three given numbers a fourth one which bears the same ratio to the third that the second does to the first is sometimes called *The Rule of Three*. It can be stated as follows:

Rule.—Multiply the means together and divide by the given extreme.

EXAMPLE 1. The ratio of 4 to 3 equals the ratio of 8 to what number?

Denote the required number by x .

Then
$$\frac{4}{3} = \frac{8}{x},$$

or
$$4 : 3 = 8 : x.$$

Now, since the product of the extremes equals the product of the means

$$4x = 24$$

and
$$x = 6, \text{ the required number.}$$

EXAMPLE 2. What is the height of a tree which casts a shadow 80 feet long at the same time a man 6 feet tall casts a shadow 5 feet long?

The ratio of the man's height to the length of his shadow is $\frac{6}{5}$, or $6 : 5$.

The ratio of the tree's height to the length of its shadow is the same.

That is, the height of the tree : 80 = 6 : 5.

Then
$$5 \times \text{height of tree} = 6 \times 80,$$

and
$$\text{the height} = \frac{6 \times 80}{5}, \text{ or } 96 \text{ feet.}$$

Exercise

1. If 12 rifles cost \$175, what will 40 rifles cost?
2. If a tree casts a shadow 208 feet long at the same time a post 4 feet in height casts a shadow $1\frac{1}{2}$ feet long, what is the height of the tree?
3. If 25 men can do a piece of work in 30 days, how long will it take 15 men to do it, working at the same rate?
4. Twenty men in 16 days can do a piece of work. How many men will it take to do it in 4 days?



ONE OF THE LARGEST AND MOST POWERFUL LOCOMOTIVES IN THE WORLD

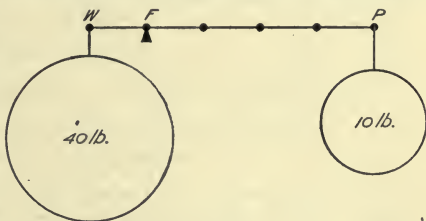
5. If a train of 30 cars carries 1100 tons, how many cars must be added to the train so that the total load may be 2800 tons?
6. If a map is drawn on a scale of 100 miles to $\frac{3}{4}$ of an inch, what distance is represented on the map by $\frac{5}{16}$ of an inch?
7. If a \$32.40 tax is paid on property assessed at \$2550, what tax should be paid on property assessed at \$5250?
8. If $6\frac{1}{2}$ tons of coal cost \$39, what will $10\frac{1}{2}$ tons cost at the same rate?
9. If in 225 pounds of milk there are 8.2 pounds of butter fat, how many pounds of milk will be required to produce 30 pounds of butter fat?

10. If 104 pounds of seed are used on a 2-acre lot, what amount will be required for $15\frac{1}{2}$ acres?
11. If 35 feet of sidewalk cost \$40.50, what will 75 feet cost if the width is the same?
12. If a 4-foot 6-inch sidewalk costs \$35, what is the cost of a 5-foot sidewalk of the same length?
13. If gun metal is composed of 2 parts of tin to 11 of copper, how many pounds of tin must be added to 190 pounds of copper for a gun-metal mixture?
14. What number bears the same ratio to $4\frac{1}{2}$ that $3\frac{1}{2}$ bears to $2\frac{1}{2}$?
15. What number bears the same ratio to 5.5 that 4.4 bears to 3.3?
16. If the standard 150-yard target has a bull's-eye 8 inches in diameter, what should be the diameter of a bull's-eye which will appear the same size at 25 yards?
17. If 84 pounds of metal are used in making 14 castings, how many pounds of metal will be used for 9 castings?

Direct and Inverse Proportions. — Such a proportion as $2:3=4:6$ is called a *direct proportion*. In the examples in the preceding exercise the proportions are all direct.

Inverse proportion can be simply illustrated as follows:

In the figure if FP is four times the length of FW , it is evident that a pressure of ten pounds at P will counterbalance a weight of forty pounds at W . That is, $\frac{10}{40} = \frac{1}{4}$, or



$$\frac{\text{Pressure}}{\text{Weight}} = \frac{\text{Length of resistance arm}}{\text{Length of pressure arm}}$$

This relation can be stated in terms of a proportion thus: The length of the pressure arm is to the length of the resistance arm as the resistance is to the pressure; or in other words, the pressure and resistance are said to be *inversely proportional* to the lengths of the lever arms to which they are applied. Relations involving inverse proportion are of very common occurrence. •

Exercise

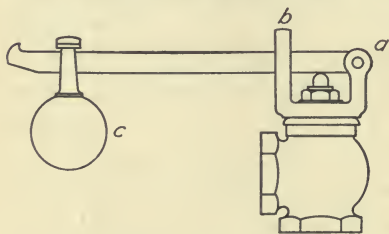
1. The arms of a lever are 8 inches and 24 inches. What load on the longer arm will balance 150 pounds on the shorter arm?

2. What weight can be lifted with a lever if the pressure is 200 pounds, the pressure arm 2.5 feet, and the resistance arm 6 inches?

3. If weights of 1 pound and 12 pounds balance on a lever and the longer arm is 3 feet, what is the length of the shorter arm?

4. A lever 12 feet long balances when weights of 50 pounds and 10 pounds are hung at its ends. Find the position of the fulcrum.

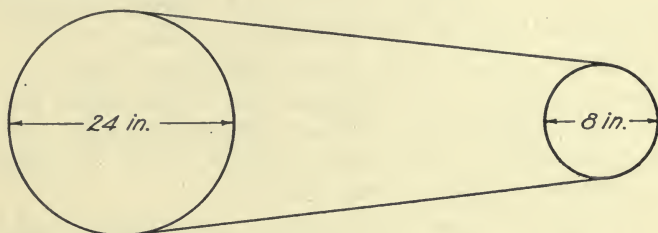
5. How much does one lift on the handles of a wheelbarrow if a 100-pound load is placed in the barrow 16 inches from the axle, and the hands 26 inches further away?



6. If an oar is 12 feet long and the rowlock is 2 feet from the handle, what force is exerted at the rowlock if the oarsman pulls with a force of 100 pounds?

7. A safety-valve lever has its fulcrum at *a*, and is to push down on the valve rod at *b* which is one inch from *a*. What must be the weight of the ball *c* if it is to be applied at a point 12 inches from *a* and exert at *b* a force of ninety pounds?

In the mechanical world are found numerous other illustrations of inverse proportion, as for example, wherever pulleys are connected by belting, or where wheels are geared together.



EXAMPLE. If a pulley 24 inches in diameter is belted to an 8-inch pulley and makes 250 revolutions per minute, how many revolutions per minute will the small pulley make?

$$\frac{\text{Diam. of large pulley}}{\text{Diam. of small pulley}} = \frac{\text{No. of revolutions per min. of small pulley}}{\text{No. of revolutions per min. of large pulley}}$$

Denote by x the number of revolutions per minute of the small pulley.

Then, $\frac{24}{8} = \frac{x}{250}$, or $24 : 8 = x : 250$

$$8 \text{ times } x = 24 \text{ times } 250$$

$$x = \frac{24 \times 250}{8}$$

$$x = 750, \text{ the number of revolutions per minute of small pulley,}$$

or the following rule can be used :

If R = revolutions per minute of large wheel
 r = revolutions per minute of small wheel
 D = diameter of large wheel
 d = diameter of small wheel

Then $R \times D = r \times d$

The solution is the same as the above.

Exercise

1. If the pulley on the headstock of a lathe is 3 inches in diameter and is belted to an 8-inch pulley on a shaft that makes 420 revolutions per minute, at what rate will a block of wood placed in the lathe revolve?

2. If the pulley on the armature shaft of a dynamo is 4 inches in diameter and is belted to a driving shaft which makes 500 revolutions per minute, what must be the size of the pulley placed on the shaft if the speed of the dynamo is to be 1800 revolutions per minute?

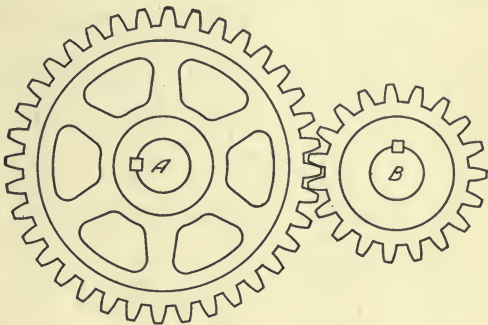
3. A shaft has upon it two pulleys, each 8 inches in diameter. The speed of the shaft is 400 revolutions per minute. What must be the size of the pulleys of two machines if when belted to the shaft one of them has a speed of 300 revolutions per minute and the other of 900?

4. A shaft which makes 300 revolutions per minute has a 6-inch pulley upon it. This pulley is belted to the pulley on a drum 3 feet in diameter. If the drum is to wind off 1000 yards of yarn per minute, what must be the size of its pulley?

5. A cycle car has a 4-inch pulley on the driving shaft belted to a 14-inch pulley fastened to a rear wheel. If the rear wheel is 26 inches in diameter, what is the speed of the car if the engine is making 800 revolutions per minute. Allow 5 per cent for belt slippage.

Gear Ratio. — The same principles as are applied to pulleys can be applied to gears. The expression *gear ratio* is frequently used to indicate the relative speeds of two connected wheels. The gear ratio of two connected wheels is the quotient obtained by dividing the number of revolutions the driving wheel makes by the number of revolutions the driven wheel makes in the same time.

EXAMPLE. If there are two gears, *A* and *B*, running together as shown in the figure, with 32 teeth in the driving wheel *A* and 18 teeth in the driven wheel *B*, how many revolutions per minute will the driven wheel make if the driving wheel makes 250 revolutions per minute?



It is obvious that the rates of two wheels geared together are inversely proportional to the number of teeth in the wheels. That is,

$$\frac{\text{Number of teeth in } A}{\text{Number of teeth in } B} = \frac{\text{Revolutions per minute of } B}{\text{Revolutions per minute of } A}$$

or,

$$\frac{\text{No. of teeth in large wheel}}{\text{No. of teeth in small wheel}} = \frac{\text{Revolutions per min. of small wheel}}{\text{Revolutions per min. of large wheel}}$$

or,

$$\frac{\text{No. of teeth in driving wheel}}{\text{No. of teeth in driven wheel}} = \frac{\text{Revolutions per min. of driven wheel}}{\text{Revolutions per min. of driving wheel}}$$

Denote by x the number of revolutions per minute of the driven wheel.

Then, $32/18 = x/250$

or $32 : 18 = x : 250$

then $18 \text{ times } x = 32 \text{ times } 250$

and $x = \frac{32 \times 250}{18}$, or 440, number of revolutions per minute of driven wheel,

or the following rule can be used :

If R = revolutions per minute of large wheel
 r = revolutions per minute of small wheel
 T = number of teeth in large wheel
 t = number of teeth in small wheel
then $R \times T = r \times t$.

The solution is the same as the above.

Exercise

1. If a 26-tooth gear is driving a 14-tooth gear, and the 26-tooth gear makes 200 revolutions per minute, at what rate is the 14-tooth gear running?

2. A 12-inch pulley running 270 revolutions per minute drives a pulley running 90 revolutions per minute. What is the diameter of the driven pulley?

3. A line shaft runs 250 revolutions per minute. A grindstone with a 14-inch pulley is to be belted to it. The grindstone is to run at 60 revolutions per minute. Determine the size of the pulley to be put on the line shaft so that the grindstone will run at the desired speed.

4. If the sprocket attached to the crank of a bicycle has 28 teeth and the rear sprocket over which the chain runs has 8, how many times does the rear wheel turn every time the pedals go round? How far does the bicycle advance in the same time if the diameter of the rear wheel is 28 inches? How many times per minute must the pedals revolve when the bicycle is going at the rate of 10 miles an hour? What is the rate of the bicycle in miles per hour when the pedals rotate 200 times per minute?

5. In a tractor demonstration test, the following were the results. Express in each case as a per cent, the ratio of the drawbar pull to the weight of the tractor.

TRACTOR	TYPE	WEIGHT	PLOWING SPEED	PLOWS	DEPTH PLOWED	DRAWBAR PULL
A	2-wheel	3380	3.5	2-14"	9.22	2705
B	4-wheel	5100	3	3-14"	8.08	1510
C	4-wheel	3250	3	3-14"	8.58	2300
D	Crawler	3500	2.5	3-14"	8.91	2455
E	Crawler	6900	2.5	3-14"	9.47	3450
F	4-wheel	5200	2.5	4-14"	8.12	3125
G	4-wheel	7800	2	4-14"	7.19	2800
H	Crawler	7400	2.25	3-14"	8.42	2600
I	3-wheel	4000	2.5	3-14"	8.31	2000



A TRACTOR-PLOWING TEST

CHAPTER XII

THE USE OF FORMULAS

A **formula** is a rule or principle expressed in algebraic symbols. Thus, the rule or principle which states that in a right triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides can be stated briefly in the form



$$c^2 = a^2 + b^2$$

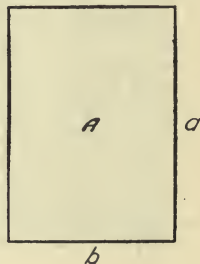
where c represents the length of the hypotenuse, and a and b the lengths of the sides. In the latter form, the rule is called a *formula*.

The rule which states that the area of a rectangle is equal to the product of the base and altitude can be expressed more briefly in a formula as follows:

$$A = a \cdot b$$

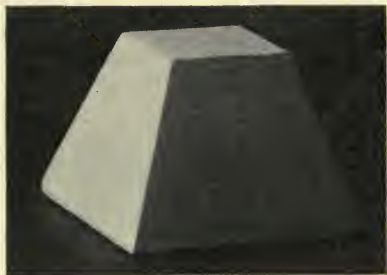
where A represents area, and a and b the sides of the rectangle.

The student has already become familiar in previous chapters with formulas for finding areas and volumes, such for example as $A = \pi r^2$, the formula for the area of a circle in terms of



the radius; and $V = \pi r^2 h$, the formula for finding the volume of a cylinder when its radius and height are given.

The great advantage of a formula is its brevity. As an illustration of this, note the simple form in which the rule can be written which states that "the volume of the frustum of a cone or pyramid is equal to the product of one third of the height by the sum of the lower base, the upper base, and the square root of the product of the areas of the bases."



FRUSTUM OF A PYRAMID

The formula reads $V = \frac{h}{3}(a + b + \sqrt{ab})$.

Evaluation in a Formula. — To evaluate in a formula is to substitute for each letter its numerical value and to perform the indicated operations.

EXAMPLE. Evaluate in $S = \frac{1}{2}gt^2$, if $g = 32.2$, and $t = 3$. Substituting in the formula the value 32.2 for g , and 3 for t , it reads:

$$\begin{aligned} S &= \frac{1}{2}(32.2)(3)^2 \\ &= \frac{1}{2}(32.2)(9) \\ &= (16.1)(9) \\ &= 144.9 \end{aligned}$$

ORAL DRILL

Evaluate in the following, using in each case the values given.

1. $A = a \cdot b$ when $a = 22$, $b = 4$

2. $A = \frac{a \cdot b}{2}$ when $a = 14, b = 20$
3. $V = a \cdot b \cdot c$ when $a = 6, b = 7, c = 3$
4. $A = \pi \cdot r^2$ when $\pi = 3.1416, r = 10$
5. $V = \frac{1}{3} \cdot b \cdot h$ when $b = 18, h = 30.5$
6. $S = \frac{1}{2} \cdot g \cdot t^2$ when $g = 32.2, t = \frac{1}{2}$
7. $d = \frac{m}{v}$ when $m = 1.2, v = \frac{1}{4}$
8. $M = m \cdot v$ when $m = 18, v = 4$
9. $F = \frac{5}{9} \cdot C + 32$ when $C = 20$
10. $C = \frac{5}{9}(F - 32)$ when $F = 50$

Exercise

Evaluate in the following, using in each case the accompanying data :

1. $V = a \cdot b \cdot c$ when $a = 14, b = 12, c = 6$
2. $A = \pi r^2$ when $\pi = 3.1416, r = 16$
3. $A = \pi r^2$ when $\pi = 3.142, r = 14$
4. $A = \frac{\pi d^2}{4}$ when $\pi = \frac{22}{7}, d = 6$
5. $S = \frac{1}{2} \cdot g \cdot t^2$ when $g = 32.2, t = 6.5$
6. $a = \sqrt{c^2 - b^2}$ when $c = 13, b = .5$
7. $A = \sqrt{s(s-a)(s-b)(s-c)}$ when $a = 8, b = 10, c = 12,$
 $s = \frac{a+b+c}{2}$
8. $V = \frac{\pi d^3}{6}$ when $\pi = 3.1416, d = 4$
9. $V = \frac{4\pi r^3}{3}$ when $\pi = \frac{22}{7}, r = 5$
10. $A = \pi(R^2 - r^2)$ when $R = 14, r = 12,$
 $\pi = 3.1416$

11. $d = \frac{1}{2}g(T^2 - t^2)$

when $g = 32$, $T = 10$, $t = 9$

12. $C = \frac{5}{9}(F - 32)$

when $F = 96$

13. $F = \frac{9}{5}C + 32$

when $C = 26$

14. $V = \frac{h}{3}(a + b + \sqrt{a \cdot b})$

when $h = 9$, $a = 16$, $b = 25$

15. $F = \frac{m \cdot v^2}{r}$

when $m = 120$, $v = 25$, $r = 15$

16. From the equations

$$s = \frac{1}{2}gt^2 \text{ and } v = gt,$$

find the time it takes a pole-vaulter to reach the ground after he has cleared the bar at 13 feet, $2\frac{1}{4}$ inches, and find the velocity with which he hits the ground on descending, if

 s = height of the bar, in feet g = attraction of gravity,
32.2 feet per second t = time in seconds to descend v = velocity in feet per second

MARK S. WRIGHT DOING 13 FT. $2\frac{1}{4}$ IN.,
WORLD'S RECORD POLE VAULT

Exercise

1. In the formula $s = \frac{1}{2}gt^2$, assume $g = 32$, and assign to t successive values from 0 to 10, and tabulate the corresponding values of s .

2. In the formula $C = \frac{5}{9}(F - 32)$, assign to F successive values from 60 to 70 and arrange in tabular form the corresponding values for C .

3. In the formula $F = \frac{9}{5}C + 32$, assign to C successive values from 20 to 30 and arrange in tabular form the corresponding values for F .

4. Make a table for corresponding values of A and r by assigning to r successive values from 0 to 10, in the formula $A = \pi r^2$.

5. Tabulate the resulting values of A obtained by assigning successive values from 1 to 10 to r in the formula $A = \frac{\pi d^2}{4}$.

Exercise

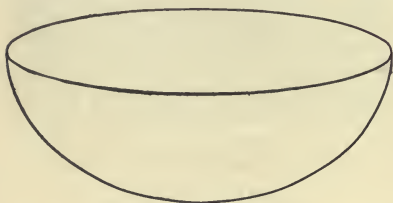
Express as formulas the following rules :

1. To find the area of a circle, multiply the square of the diameter by .7854.

2. To find the area of a sphere, multiply the square of the diameter by 3.1416.

3. To find the volume of a sphere, multiply the cube of the diameter by .5236.

4. To find the volume of a spherical segment, add the square of its altitude to three times the square of the radius of the base ; multiply this sum by the altitude, and the product by .5236.



SPHERICAL SEGMENT

5. To find the volume of a cylinder, multiply the area of the base by the altitude.

6. To find the volume of a cone or pyramid, multiply the area of the base by one third the altitude.

7. To find the volume of a frustum of a cone or pyramid : To the sum of the areas of the bases add the square root of their product and multiply the sum by one third the altitude of the frustum.

8. To find the area of an ellipse, multiply the product of the diameters by .7854.

9. To find the area of an equilateral triangle, multiply the square of one side by .433.

10. To find the diameter of a circle, multiply the circumference by .3183.

11. To find the radius of a circle, multiply the circumference by .15915.

12. To find the side of a square inscribed in a circle, multiply the diameter by .7071.

13. To find the side of a square inscribed in a circle, multiply the circumference by .2251.

14. To find the side of a square equal in area to that of a given circle, multiply the diameter by .8862.

15. To find the side of a square equal in area to that of a given circle, multiply the circumference by .2821.

16. To find the horse power of an engine, multiply together the mean effective pressure on the piston in pounds per square inch, the length of the stroke in feet, the area of the piston in square inches, and the number of strokes per minute; divide this product by 33,000.

17. To convert Centigrade reading to Fahrenheit, multiply by 1.8 and add 32.

18. To convert Fahrenheit reading to Centigrade, subtract 32 and multiply by .56.

19. To find the area of a triangle when the sides are given, multiply in succession half the sum of the sides by the three remainders obtained by subtracting each side separately from the half-sum of the sides, and take the square root of this product.



CENTIGRADE AND
FAHRENHEIT
THERMOMETERS



BEAN THRASHER IN OPERATION

CHAPTER XIII

AGRICULTURAL PROBLEMS

1. If three fourths of the weight of potatoes is water, and if a bushel of potatoes weighs 60 pounds, what is the weight of water in 300 bushels of potatoes? What is the weight of that part which is not water?
2. If wheat weighs 60 pounds per bushel, and the average yield is 28 bushels per acre, what will be the weight of the wheat grown on a field of 40 acres?
3. If during a rainstorm 0.72 in. of rain fell on an 80-acre field, how many cubic inches of water fell on 1 square foot? on 1 square rod? on 1 acre? on the whole field?
4. If a kitchen poorly arranged requires the housekeeper to take 100 more steps, 20 inches to a step, each day in preparing the meals, than she would in a well-arranged kitchen, how many miles of unnecessary travel does this amount to in the course of a year?

5. If a flock of 60 hens averages 95 eggs a year each, what is the value of the eggs at 75 cents per dozen?

6. What is the difference in the value of two hens, one laying 190 eggs a year and the other 90 eggs a year, if the average price of eggs is 75 cents per dozen?

7. It is estimated that on the farm hens average 90 eggs a year each, but by careful selection an average of 140 eggs a year each could be obtained. What would this difference amount to for a year with a flock of 100 hens with eggs at 75 cents a dozen?

8. A flock of 40 hens having a barnyard for a run are fed daily 3 lb. of wheat worth \$2 a bushel. If the hens average 90 eggs a year each when eggs are worth 75 cents a dozen, what is the profit on the flock?

9. A man paid out \$66 in four weeks for feed for 360 hens. He sold 744 eggs at 60 cents a dozen during the first two weeks, 564 eggs at 75 cents a dozen during the third week, and 660 eggs at 85 cents a dozen the fourth week. His receipts were what per cent greater than the cost of the feed?

10. If a cow averages daily 3 gallons of milk which is 4 per cent butter fat, how much butter does she produce per year?

(One gallon of milk weighs about 8.5 lb. One pound of butter fat will make about $1\frac{1}{8}$ lb. of butter.)

11. For \$50 a man can buy a scrub cow that gives 14 lb. of milk per day, or for \$120 a Jersey that gives 25 lb. of milk per day. If the first cow's milk is 2 per cent butter fat and the Jersey's $4\frac{1}{2}$ per cent, and the butter averages 70 cents a pound the year round, which cow will yield the greater per cent on the investment the first year?

12. A farmer spent on a 12-acre field the following amounts: plowing, \$30; harrowing and rolling, \$22; seed, \$16; drilling, \$2.75; cutting and threshing, \$27. His income from the field was 240 bushels of wheat at \$2 a bushel, and \$5 for pasturage. Find the balance in his favor.

13. A man had 8 acres planted to asparagus. The expense per acre for fertilizer was \$ 40, for labor before cutting, \$ 22, for cutting and bunching, \$ 42. From each acre he cut 2000 bunches and sold them at 10 cents a bunch, allowing a commission of 3 % for selling. What was his net return from the 8 acres ?

14. If a bushel of wheat weighs 60 lb., how many pounds of wheat will be produced on a field of 40 acres where the average yield is 24 bu. to the acre ?

15. After a heavy summer rain the water of a small stream was found to contain 1 lb. of sediment for every 500 gallons of water. If the rainfall was 1 in., the area of the basin drained 4 square miles, and the amount of water that ran off $\frac{1}{4}$ of what fell on the ground, how much soil did the rain carry away ?

16. If the water running from a piece of planted land contained 1 lb. of sediment for every 250 gallons of water, how much soil was carried away from a 40-acre field after a 2-inch rainfall, $\frac{1}{4}$ of the water running off ?

17. If seed corn averages 800 grains to the ear, how many ears will it take to plant an acre, 2 grains to the hill, when the rows are 3 ft. 8 in. apart, the hills in the row being the same distance apart ?

18. If a farmer can increase the weight of each ear of corn 2 ounces by proper selection of seed, what will the increase amount to on a 40-acre field averaging 6500 stalks per acre, 1 ear for every stalk, when corn is selling for 95 cents per bushel ?

19. If 12 ears of selected seed corn will plant an acre, how many ears will it take to plant a field 40 rods by 60 rods ?

20. If a solution for spraying fruit trees and plants is made up of

Lime	4 lb.
Copper sulphate	4 lb.
Paris green	$\frac{1}{4}$ lb.
Water	50 gal.



SPRAYING OUTFIT

what will 100 gallons of such a mixture cost if lime costs 25 cents a pound, copper sulphate 40 cents a pound, and Paris green 75 cents a pound?

21. What is the cost per tree for the mixture of Ex. 22 if a sufficient amount to spray one tree is $2\frac{1}{2}$ gallons?

22. If a solution for spraying is prepared, without the Paris green, but otherwise like the above, what will it cost per 100 gallons if sulphate of copper costs 40 cents a pound and lime 25 cents a pound?

23. For a vineyard of 30 acres how many pounds of sulphate of copper and lime are necessary if 150 gallons of mixture will spray 1 acre? (See Ex. 22.)

24. If a man with a team at \$6 a day can spray 4 acres in a day, what will it cost to spray 40 acres of grapes if the spraying mixture costs \$6 per 100 gallons?

25. The estimated value of a crop of grapes is \$450 per acre. Thorough spraying costs \$9 per acre. If, without spraying, 30 per cent of the crop is lost, what is the net value of spraying 10 acres?

26. If 100 gallons of a lime-sulphur mixture for spray contains 40 lb. of lime and 30 lb. of sulphur, what will it cost for sufficient to spray 300 trees, 25 gallons being required for 9 trees, if sulphur is worth 25 cents per pound, lime 25 cents per pound, and the labor of preparing the mixture 75 cents per 100 gallons?

27. How many pounds of Paris green are needed to spray 300 trees if 5 gallons of spray are used to a tree and every 150 gallons contain 1 lb. of Paris green?

28. What is the value of a 160-acre cotton-crop if two bales are raised to the acre and cotton is worth \$.40 a pound? A bale of cotton weighs 500 lb.

29. What is the value of the annual alfalfa crop on 160 acres if 165 tons can be raised on a 10-acre piece and alfalfa hay is worth \$22 a ton?

30. Determine the relative costs of fencing two 40-acre fields if one of them is square and the other is a rectangle whose length is twice its width.

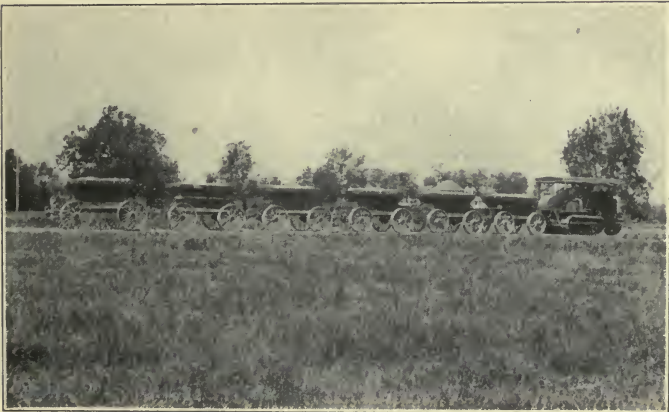
31. If it requires a pull of 90 pounds to draw a ton over a good gravel road, and 2.4 times as much to draw the same load over a common dirt road, what pull is required for the dirt road?

32. The gross returns from a 10-acre orange grove were \$ 4560. The expenses were \$ 922. What was the net return per acre ?

33. The yield for oranges by years is as follows :

4th year		30 packed boxes
5th year		60 packed boxes
6th year		100 packed boxes
7th year		150 packed boxes
8th year		175 packed boxes
9th year		200 packed boxes

Present the foregoing data in graphic form.



HAULING BY TRACTOR

34. Find the net profit per cow in a herd of 18 if the total received from cream in one year was \$ 1902, and the expense of keeping the herd was as follows :

Hay		\$ 324
Silage and roots		72
Bran and oil meal		144
Pasturage for 7 mo.		126

35. Put in the form of a graph the following figures, giving the classification of farms in a certain state, with reference to acreage :

19 acres or less	25 per cent
20 to 49 acres	23 per cent
50 to 99 acres	12 per cent
100 to 174 acres	14 per cent
175 acres or more	26 per cent

36. Determine from the following figures the average number of pounds of milk per cow per year, and the total number of pounds of butter fat if the average butter test was .344.

MONTHS	NUMBER OF COWS	POUNDS OF MILK
November	12	6000
December	9	4500
January	8	4008
February	7	4078
March	11	8690
April	11	9530
May	13	11,622
June	14	13,004
July	15	13,710
August	16	14,669
September	16	12,403
October	16	11,388

37. Find the net return per acre for one and two-thirds acres of peach trees if the outlay and income account was as follows :

Plowing and cultivating	\$ 15
Pruning	30
Thinning	30
Picking 14 tons at \$ 2.50 per ton	35
Drying	50
Gross returns for the sale of the fruit	375

38. Find the net return on a raspberry crop which yielded 500 crates to the acre if they sold at \$1.50 per crate, and the expense account was as follows :

Fertilizer	\$ 27.50
Cutting canes	10
Sprouting	6
Hoeing	8
Cultivating and training canes	10
Picking	200
Crates	80
Assistance in handling	30



FRUIT DRYING AT SANTA CLARA, CALIFORNIA

39. Present in the form of a graph the following data, showing the number of fruit trees in a California county :

Apple	51,000 trees	Peach	654,000 trees
Apricot	588,000 trees	Pear	147,000 trees
Cherry	393,000 trees	Plum	295,000 trees
Fig	3,000 trees	Prune	5,690,000 trees
Nectarine	1,500 trees	Other kinds . . .	430,000 trees
Olive	17,000 trees		

40. The following amounts were received from the sale of milk for one year: Jan., \$65.50; Feb., \$74.20; Mar., \$76.80; Apr., \$78.25; May, \$85.30; June, \$90.50; July, \$78.20; Aug., \$73.80; Sept., \$71.80; Oct., \$69.10; Nov., \$61.50; Dec., \$63.40. Find the amount received during the year.

41. Two 40-acre fields were sown in oats, the average yield of which was 45 bushels to the acre. What was the value of the crop at \$.80 a bushel?

42. If each hen laid on an average 100 eggs per year, what was the average monthly return if the eggs were sold for 85 cents a dozen?

43. Two boys were asked to figure out the number of posts necessary to build a fence 200 feet long, the posts to be placed 8 feet apart. One boy said it would take 25 posts, the other said it would take 26. Was either boy's answer correct?

44. The same boys were asked how many posts would be necessary for a fence around a corral 160 feet by 200 feet, the posts to be placed 8 feet apart. Both boys said it would take 91 posts. Was their answer correct?

45. In the following, fill in the columns of net weights, totals, and values.

TICKET SHOWING WEIGHTS OF HAY

GROSS WEIGHT	WEIGHT OF WAGON	NET WEIGHT	VALUE AT \$27 50 A TON
3676	1470	—	—
3862	1490	—	—
4140	1510	—	—
Totals			

46. A field 180 ft. by 200 ft. is inclosed by four lines of galvanized iron wire. Eight yards of this wire weigh a pound, and the wire cost 8 cents a pound. What was the cost of the wire?

47. How many acres are there in a rectangular piece of land 8450 ft. long and 3580 ft. wide ?

48. How many cords of wood are there in a pile 80 ft. long, 6 ft. high, and 4 ft. wide ?

49. A man bought a farm for \$ 3500 and sold it for \$ 4200. If the buying and selling were done by a real estate agent who charged 5 % for each transaction, what per cent of the purchase price did the man gain ?

50. If the weight of oak ashes is .03 of the weight of the wood burned, and the weight of carbonate of potash contained in the ashes is .065 of the weight of the ashes, how many pounds of carbonate of potash are there in the ashes of 1 cord of oak, if stacked oak wood weighs 2 tons per cord ?

51. A farmer sold a team of mules for \$ 550 but did not receive his pay for them until 1 yr. 8 mo. after the sale. He had at the same time a cash offer of \$ 505 for them. Did he gain or lose by the sale and how much, money being worth 6 % ?

52. Find the value of the potato crop on 60 acres, if the average yield is 130 sacks to the acre and the potatoes are worth \$ 4.20 per sack.

53. A farmer shipped 50 sacks of potatoes to a commission merchant who sold them at \$ 4.95 a sack, and charged a commission of 5 %. What did the farmer receive ?

54. A farmer shipped 22 tons of hay to a commission merchant who sold it at \$ 18.50 per ton. How much did the merchant remit to the farmer, after deducting his commission of 5 % ?

55. A farmer shipped 1140 pounds of butter to a commission merchant, who sold it at 70 cents a pound. If the cost of shipping was \$ 2.50 and the cartage amounted to \$ 1.50, how much did the shipment net the farmer, after paying a commission of 5 % ?

56. A certain school district voted \$15,000 to erect a new schoolhouse. The assessed valuation of the property in the district was \$750,000. Find the rate of taxation.

57. If newly harvested potatoes lose in weight 1% a month for about five months, and the price of potatoes in October



RIVER TRANSPORTATION

was \$2.45 and in the following March was \$2.80, did it pay to hold the crop till March before marketing? How much was gained or lost by holding the crop?

58. Prepare a ticket similar to that in Ex. 45 for 6 loads of coal whose weights are as follows: Gross weights, 4540, 4664, 4860, 5280, 4970, 5130; weights of wagons: 1660, 1652, 1540, 1490, 1475, 1650. Price of coal \$14.25 per ton.

CHAPTER XIV

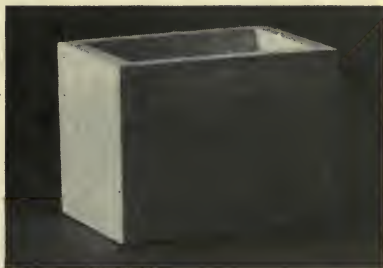
MECHANICAL PROBLEMS

1. In a 10-lb. sample of solder composed of 5 parts zinc, 2 parts tin, and 1 part lead, how many pounds of each metal are there?

2. Find the cost at 8 cents a pound of a concrete tank 3 ft. long, 2 ft. wide, and 2 ft. high, the sides and bottom of the tank to be 2 in. thick.

3. What will the tank of Ex. 2 cost if made of cast iron, at the same price per pound?

4. How many tanks 3 ft. 4 in. long, 2 ft. 4 in. wide, and 2 ft. 2 in. high can be made from 1000 cu. yd. of concrete if the sides and bottom of the tank are $2\frac{1}{2}$ in. thick?



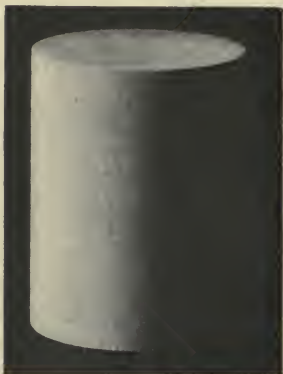
HOLLOW PRISM

5. Find the cost at 8 cents per pound of 1000

concrete receptacles in the form of the frustum of a pyramid 3 ft. square at the top, 2 ft. 8 in. at the bottom, outside measurements, 2 ft. 2 in. high, the sides and bottom of the boxes to be 2 in. thick. Find the cost at 25 cents per cubic yard of sufficient earth to fill them all.

6. If an engine pumps $2\frac{1}{4}$ gallons of water per stroke and makes 46 strokes per minute, how many gallons will it deliver in an hour?

A convenient formula for finding capacity of cylindrical tanks standing on end is the following: $V = .0034 d^2 h$



CIRCULAR CYLINDER

where V = capacity in gallons

d = diameter in inches

h = depth in inches

7. Find the capacity in gallons of an upright cylindrical tank 24 in. in diameter, 30 in. high.
8. Find the capacity of an upright cylindrical tank 4 ft. in diameter and 6 ft. high.
9. What is the capacity in gallons of a cylindrical can 15 in. in diameter and 23 in. high? What weight of materials weighing 50 lb. per cubic foot will it hold?
10. What is the height of a can of the same diameter as the can of Ex. 9 which will hold about 100 lb. of the same material?
11. What are the dimensions of a cylindrical tank whose height and diameter are equal, if the capacity of the tank is 75,000 gallons?
12. Find the cost of lumber and nails to inclose a corral 120 ft. by 200 ft. with a 4-rail fence 5 ft. high, according to the following specifications: posts to be 4 in. by 4 in., 8 ft. apart, and set 2 ft. in the ground; rails to be 1 in. by 6 in., fastened with 8-penny nails, 2 in each rail at each post; cost of lumber \$ 36 per M; nails 35 cents per pound. (There are about 106 8-penny nails to the pound.)
13. Determine the cost of materials and labor to inclose a 10-acre field in the form of a square with a 3-rail fence 4 ft. high, post holes to cost 10 cents apiece, and labor \$ 3 a day, assuming one man can build 200 ft. of fence per day; other specifications as in Ex. 12.

14. With lumber at \$35.50 per M find the cost of materials for a coal bin with sides and bottom, no top, 6 ft. square and 3 ft. deep; corner posts 4 in. by 4 in.; floor to be laid on stringers 2 in. by 6 in., placed 3 ft. apart.

15. If a stick of timber is 16 in. square, what distance must be laid off from each corner to line for a chamfer, if the stick is to be octagonal?

16. If steel expands 0.000007 of its length for each degree increase in temperature (Fahrenheit), how much will a 30-ft. steel rail increase in length when heated from 32° to 120° ?

17. A cornucopia has a circular top $2\frac{3}{4}$ inches across. It is $9\frac{1}{2}$ inches deep. How many cubic inches will it contain?

18. A ball 9 inches in diameter is placed in a cubical box 10 inches on an edge, inside measurement. How many cubic inches of the box are not occupied by the ball?

19. How many loads of earth will have to be excavated to make a cut for a flume of rectangular section 300 feet long, 16 feet wide, and 8 feet deep?

20. Find the volume of a sphere 1 foot in diameter; of a cylinder of the same diameter and altitude; and of a cone of the same diameter and altitude. The volume of the sphere is how many times the volume of the cone? The volume of the cylinder is how many times the volume of the cone?



SPHERE

21. If the cost of operating a motorcycle is \$0.00 $\frac{3}{4}$ per mile, how far can one ride for a cent? for \$.75? What is the cost per thousand miles?

22. The driving pulley on a shaft is 42 in. in diameter and makes 200 revolutions per minute. How many revolutions per minute will the driven pulley make if its diameter is 3 ft. ?

23. The diameter of the driving pulley is 9 in. and its speed is 1000 revolutions per minute. At what speed will the driven pulley run if its diameter is 4 in. ?

24. If a locomotive driving wheel is 6 ft. in diameter and makes 110 revolutions per minute, find the speed of the locomotive in miles per hour.

The horse power of a 4-cycle gas engine is given by the formula :

$$\text{HP} = \frac{(\text{Diam. of Cylinder})^2 \times (\text{Length of Stroke}) \times (\text{No. of Rev. per Min.})}{18000 \times (\text{No. of Cylinders})}$$

25. Find the horse power of a cycle-car gas engine with 2 cylinders, $3\frac{1}{2}$ -in. bore, $3\frac{5}{8}$ -in. stroke, running at 2500 revolutions per minute.

26. Find the horse power of a single-cylinder engine with 3-in. bore, $3\frac{1}{2}$ -in. stroke, running at 800 revolutions per minute.

27. Brass contains 65 per cent copper and 35 per cent zinc. When copper is worth 40 cents a pound and zinc 8 cents, what ought brass castings to cost if 2 cents a pound is allowed for casting the brass ?

28. If 400 cubic feet of stacked hay weighs one ton, find the weight of a stack 22 ft. by 40 ft. by 24 ft.

29. The material excavated from a cellar 52 ft. long by 34 ft. wide and 6 ft. deep was evenly distributed over a lot 30 ft. by $137\frac{1}{2}$ ft. To what depth was the lot covered ?

30. Find the cost at 30 cents per load to excavate the ground for a cellar 24 ft. wide and 42 ft. long, the ground being excavated to a depth of 7 ft. 6 in. at one end and 2 ft. 6 in. at the other.

31. A rectangular piece of tin 16 in. by 20 in. has four equal squares cut out at the corners, and the sides and ends are then

turned up so as to form a rectangular tray. What is the capacity in pints of the tray if the sides of the square are 1 in.? if $1\frac{1}{2}$ in.? if 2 in.? if $2\frac{1}{2}$ in.? if 3 in.? if $3\frac{1}{2}$ in.? if 4 in.? Tabulate your results. What depth gives the maximum capacity?

32. If the piece of tin is 14 in. by 24 in. and other conditions as in the preceding example, determine the capacity of the tray if the sides of the squares are 1 in., 2 in., 3 in., 4 in.

33. A concrete swimming-tank is 40 ft. long and 22 ft. wide. Its depth is 10 ft. at one end and 4 ft. at the other. How many gallons of water are in it when it is filled to within 1 ft. of the top? (One cubic foot is equivalent to about 7.5 gallons.)

34. Cast iron disks are to be designed for a dumb-bell whose weight can be varied. The disks are to be $4\frac{1}{2}$ in. in diameter with a $1\frac{3}{8}$ -in. hole through the center, and are to weigh 3 lb., 4 lb., and 5 lb. What must be the thickness of each?

Exercise

1. Given the following Paving Material data:

A Paving Mixture consists of:

1 part of cement,

3 parts of sand,

6 parts of crushed rock.

A sack of cement is equivalent to about 1.3 cubic feet,

A wheelbarrow load of sand to about 4 cubic feet,

A wheelbarrow load of rock to about 4 cubic feet.

If all the materials necessary for the mixture are to be dumped every 30 feet, how much of each material should be dumped at each place if the street is 40 feet wide, and the paving mixture is to be laid 5 inches deep?

2. How much of each material at each place if the street is 50 feet wide?

3. How much if the street is 30 feet wide?



ROAD CONSTRUCTION

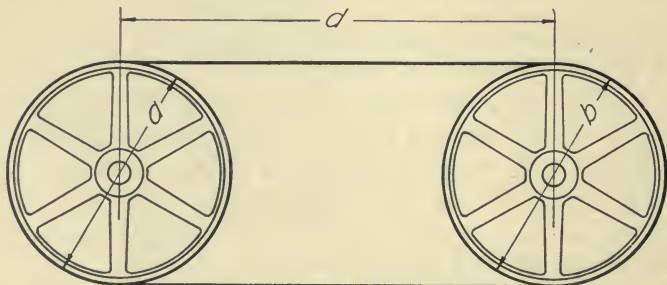
Pulleys and Belting.— In the following formulas for length of belting,

l = length of belt in feet

a = diameter of larger pulley in inches

b = diameter of smaller pulley in inches

d = distance between centers in feet



OPEN BELT PULLEYS SAME SIZE

I. For open belt where the difference in the diameters of the pulleys is not 'great,

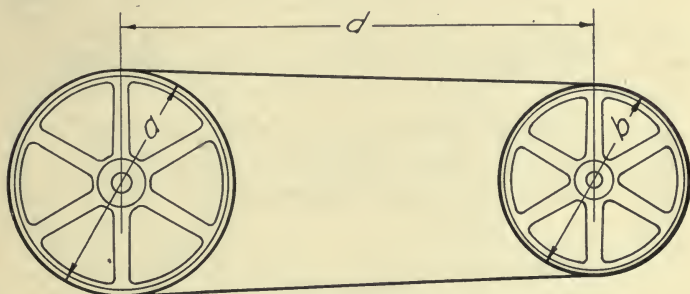
$$l = .13(a + b) + 2d$$

EXAMPLE 1. Two 16-inch pulleys, 10 ft. between centers, are connected by an open belt. What is its length?

Here $a = 16$ in., $b = 16$ in., and $d = 10$ ft.

Then $l = .13(16 + 16) + 2(10)$
 $= .13(32) + 20 = 4.16 + 20 = 24.16$ ft.

EXAMPLE 2. Two pulleys, 24 in. and 20 in. in diameter, are 18 ft. between centers. What length of belting is necessary to connect them?



OPEN BELT PULLEYS DIFFERING SLIGHTLY IN SIZE

Here $a = 24$ in., $b = 20$ in., and $d = 18$ ft.

Then $l = .13(24 + 20) + 36$
 $.13(44) + 36 = 41.7$ ft.

Exercise

1. The distance between the centers of two 24-in. pulleys is 18 ft. What must be the length of the connecting belt?

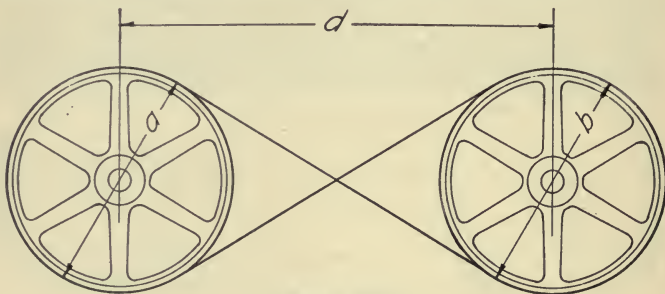
2. Two pulleys belted together are 20 in. in diameter. The distance between their centers is 22 ft. What is the length of the belt?

3. Two pulleys, the distance between whose centers is 14 ft., are 18 in. in diameter. If the pulleys are both replaced by 22-in. pulleys, how much must the belt be lengthened?

4. What length of belting is required to connect two pulleys, 24 in. and 22 in. in diameter, the distance between whose centers is 20 ft.?

5. Two pulleys have their centers 18 ft. apart; their diameters are 26 in. and 24 in. What length of belt is required to connect them?

6. If the pulleys in Ex. 5 are replaced by 22-in. pulleys, what change must be made in the length of the belt?



CROSSED BELT

II. For crossed belt, or for open belt where the difference in the diameters is great,

$$l = .14(a + b) + 2d$$

EXAMPLE 1. What length of crossed belt is required to connect two 16-inch pulleys, 12 ft. between centers?

$$\begin{aligned} l &= .14(16 + 16) + 2(12) \\ &= .14(32) + 24 = 4.5 + 24 = 28.5 \text{ ft.} \end{aligned}$$

EXAMPLE 2. What length of open belt is required to connect two pulleys of diameters 16 in. and 8 in. with centers 10 ft. apart?

$$\begin{aligned} l &= .14(16 + 8) + 2(10) \\ &= .14(24) + 20 = 3.4 + 20 = 23.4 \text{ ft.} \end{aligned}$$

Exercise

1. Two pulleys are 16 ft. between centers. The pulleys are 24 in. and 12 in. in diameter. What length of belt is required?
2. The distance between the centers of a 28-in. pulley and an 18-in. one is 18 ft. What length of belt is required?
3. Two pulleys 30 in. and 18 in. in diameter, are 16 ft. between centers. If the 30-in. pulley is replaced by a 28-in. one, how much must the belt be shortened?
4. The distance between centers of two 22-in. pulleys is 20 ft. How long must the connecting belt be if crossed?
5. Two pulleys connected by crossed belting are 20 in. in diameter. The distance between centers is 24 ft. What length of belt is required?
6. Two 16-in. pulleys whose distance between centers is 18 ft. are connected by crossed belting. If the pulleys are both replaced by 20-in. pulleys, how much must the belt be lengthened?
7. What is the length of the connecting crossed belt if the distance between centers is 22 ft. and the pulleys are 24 in. and 18 in. in diameter?
8. The distance between the centers of a 28-in. pulley and an 18-in. one is 20 ft. How much more belting will be required if crossed than if not crossed?
9. Two pulleys connected by crossed belting are 30 in. and 16 in. in diameter. Their distance between centers is 16 ft. If the 16-in. pulley is replaced by a 12-in. one, how much must the belt be lengthened?
10. In the preceding exercise, if the pulleys are replaced by two 18-in. ones, what change will be necessary in the length of the belt?

LENGTH OF BELTING IN A COIL

 l = length of belting, in feet d = diameter of roll, in inches e = diameter of eye, in inches n = number of turns in the coil

$$l = .13 n(d + e)$$

EXAMPLE. The diameter of a coil of belting is 19 in., the diameter of the eye is 5 in., and the number of turns is 14. Determine the length of belting in the coil.

$$\begin{aligned} l &= .13(14)(19 + 5) \\ &= .13(14)(24) = 44 \text{ ft.} \end{aligned}$$

Exercise

1. How many feet of belting in a coil of 30 turns whose diameter is 21 in. and whose eye is 6 in.?
2. How much belting in a roll of 41 coils, 2 ft. in diameter, with a 9-in. eye?
3. What is the length of belting in a solid roll of 40 coils, 2 ft. in diameter?
4. At 30 cents per running foot, what is the cost of a roll of belting of 16 turns, with outside diameter 2 ft., inside diameter 20 in.?

The length of rope, wire, hose, etc., wound on a reel is given by the formula :

$$l = .13 nt(a - e)$$

where

n = no. of coils side by side

t = no. of layers in coil

a = diameter over all

e = diameter of shank of reel

Exercise

1. How much $\frac{1}{2}$ -in. rope can be wound on a reel with the dimensions given in the figure?

2. How much 1-in. garden hose can be wound on the reel of Ex. 1? How much $\frac{1}{8}$ -in. carpenters' chalk-line?

3. How much wire .05 in. in diameter can be wound on a reel 11 in. in diameter over all, with a shank 6 in. in diameter and 6 in. long? How much wire if the diameter is twice as great?

4. If the tire of a cart wheel is lengthened 2 feet, how much clearance will there be?

5. How many feet of $\frac{1}{2}$ -in. wire cable will a reel hold if it has a shank 4 ft. long, 1 ft. in diameter, and a 3-in. flange?

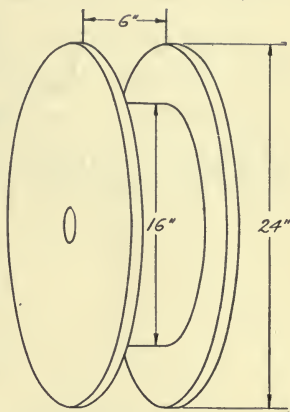
6. If a captive balloon is held by a wire cable $\frac{1}{4}$ in. in diameter which weighs .1 lb. per foot and has a tensile strength of 2400 lb., how high can the balloon be allowed to go without danger of the cable's breaking of its own weight?

7. What must be the inside dimensions of a cubical box which will hold 8000 quarter-inch buckshot?

8. If the 8000 buckshot of Ex. 7 are melted and recast in the form of a cylinder whose diameter and height are equal, what is its diameter and height if a 5 per cent allowance is made for waste in melting?

9. If the cylinder of Ex. 8 is melted and recast in the form of cubical blocks 1 in. on each edge, how many can be made, allowing 5 per cent for waste?

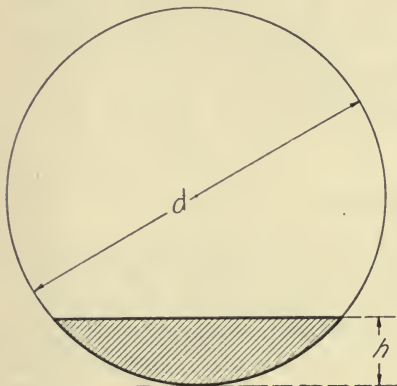
10. Determine the weight per foot of lead pipe 2 in. in diameter and $\frac{1}{8}$ in. thick. (A cubic foot of lead weighs 710 lb.)



11. What must be the depth of a 5-gallon pail the diameter of whose top is 14 in. and whose base is 9 in.?

12. To what depth will the pail of Ex. 11 be filled when it holds 4 gallons? To what depth when it holds 3 gallons?

13. A pail is 8 in. in diameter at the top, 6 in. in diameter at the bottom, and 7 in. deep. Where on the side must it be marked to indicate 1 qt.? 2 qt.? 3 qt.?



CIRCULAR SEGMENT

14. A cylindrical tank is 4 ft. long and 2 ft. 6 in. in diameter, inside dimensions. When it stands on end the water in it is 18 in. deep. How deep will the water be when the cylinder is lying on its side? (See below.)

15. A certain cubical tank will hold 216 cu. ft. of water. What must be the dimensions of a similarly shaped tank which will hold 300 gallons?

The area of a circular segment can be found by the formula

$$A = \frac{4h^2}{3} \sqrt{\frac{d}{h} - 0.608}$$

where

A = area

d = diameter of circle

h = height of segment

16. Using the formula just given, find the area of the segment if $d = 16$ inches and $h = 6$ inches.

The contents of a partially filled horizontal cylindrical tank can be computed from the formula

$$C = .005772 \, h l \sqrt{\frac{d}{h} - 0.608}$$

where

C = contents in gallons

d = diameter in inches

h = depth of liquid in inches

l = length of tank in inches

17. Find the number of gallons of liquid in a horizontal cylindrical tank 6 feet long, 16 inches in diameter, if filled to a depth of 6 inches.

CHAPTER XV

MISCELLANEOUS PROBLEMS

1. A man who owned $\frac{5}{8}$ of the stock in a certain company sold $\frac{3}{5}$ of what he owned. What part of the company stock did he then own?

2. An automobile is bought for \$1600, run for 4 years and sold for \$750. During that time the cost for gasoline and oil averaged \$.75 a day; repairs, including tire expense, amounted to about \$25 a month; license \$15 a year; and insurance \$25 a year. What did the automobile cost per day?

3. A map of Kansas is made on a scale of 1 inch to 100 miles. The map measures 4 inches by 2 inches. Find from it the area of the state.

4. If a circus ring is 42 feet in diameter and a horse makes 23 lopes in going round it, how far does the horse go in each lope? Assume that the path of the horse is 6 inches inside the rim.

5. Prepare a diagram setting forth graphically the following facts. Of the persons who have attained sufficient distinction to be listed in "Who's Who in America" 71 % were college trained; 16 % had high school training; 9 % stopped after the grammar school; 3.8 % were prepared by private tutors; and .2 % are self-made.

6. A tank 8 feet 3 inches long, 3 feet 6 inches wide, and 4 feet 6 inches deep is $\frac{2}{3}$ full of water. How many gallons are there in the tank?

7. How many board feet are there in a stick of timber 18 feet long, 16 inches wide, and 12 inches thick?

8. What is the cost of a casting weighing 240 pounds at $6\frac{1}{2}$ cents a pound?

9. The side of a square is 100 feet. Find the length of a diagonal.

10. Divide 60 into two parts in the ratio of 2 to 3.

11. What per cent of 3 inches is $\frac{3}{4}$ of an inch, $\frac{3}{16}$ of an inch, $\frac{11}{2}$?

12. How many feet of wire will be required to fence (three wires) a 12-acre field if it is square? How many if it is twice as long as it is wide?

13. What is the value of a ton of silver dollars at \$ 1.30 an ounce if each dollar weighs $412\frac{1}{2}$ grains?

14. If pocket knives are worth \$21 a dozen, what is the price per knife?

15. On a map made on a scale of 6 inches to a mile, a rectangular field is represented by a space 1 inch long and $\frac{1}{4}$ of an inch wide. How many acres are there in the field?

16. A machinist receiving \$6.25 per day is given a 10 per cent increase. Later he is cut 10 per cent. What is his wage then? Why is it not the same as it was before the increase?

17. Find the cost of making a driveway 200 yards in length and 8 yards wide, the soil being first excavated to a depth of 14 inches at a cost of 25 cents per cubic yard; crushed stone then being put in 8 inches deep at a cost of 30 cents per cubic yard, and gravel placed on top 6 inches deep at a cost of 50 cents per cubic yard.

18. If lubricating oil can be bought at 22 cents a quart, how many gallons will \$5 buy?

19. Divide 66 into parts proportional to 2, 3, and 4.

20. How many 5-ounce bottles can be filled from 5 gallons of liquid?

21. What is the weight of \$1,000,000 in silver if one dollar weighs $412\frac{1}{2}$ grains?

22. What are the total box-office receipts for a single performance if 1000 seats sell for \$ 2 each, 2000 at \$ 1 each, 4000 at \$.75 each, and 5000 at \$.50 each ?

23. Find the volume and convex surface of a cone if the diameter of the base is 16 inches and the altitude is 18 inches.

24. How many eggs at 85 cents a dozen can be bought for \$ 5 ?

25. What will it cost to excavate a roadway to a depth of a foot and a half at 50 cents per cubic yard, if the width is 50 feet, and the length is 1000 feet ?

26. What will curbing for both sides of the roadway mentioned in the preceding example cost at 25 cents per linear foot ?

27. When a man runs 100 yards in 10 seconds, what is his rate in feet per second ? What is his rate in miles per hour ? At the same rate how long would it take him to run a mile ?

28. If a bushel of wheat weighs 60 pounds, how many bushels are there to the ton ?

29. How many 8-ounce packages of sulphur can be put up from one ton of it ?

30. If sound travels 1100 feet per second, how far distant was a rifle whose discharge was heard 4 seconds after firing ?

31. If 110 high school seniors adorn themselves with ribbon streamers in the class colors, red and white, $\frac{3}{4}$ of a yard of each, what will be the total cost of the ribbon at 20 cents a yard ?

32. How deep must a wagon-bed 4 feet wide and 8 feet long be loaded with wet sand (specific gravity 2) to weigh 2 tons ?

33. How many conical glasses 4 inches in diameter and 3 inches deep can be filled with a gallon of liquid ? How many gallons of liquid will it take to fill 100 glasses ?

34. A quarter of a ton of bronze castings are to be made. The material is 77 per cent copper, 8 per cent tin, and 15 per cent lead. How many pounds of each metal are required?

35. When a man runs 100 yards in $9\frac{2}{5}$ seconds, how many feet does he travel in one second? What is his rate in miles per hour?

36. If a bottle and a cork together cost \$1.10 and the bottle costs \$1 more than the cork, what is the cost of the cork?

37. If an automobile can be driven 28.7 miles on a gallon of gasoline, how many gallons are consumed in a 1000-mile trip?

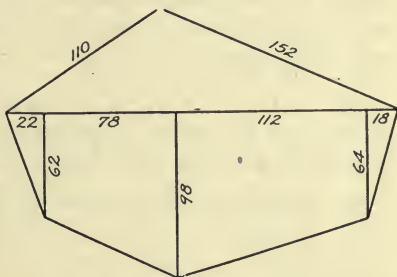
38. Two men start in business with a capital of \$7500. One of them furnishes \$4000 and the other \$3500. At the end of a year the profits are \$3250. How much is each man's share?

39. If a chrome-steel armor plate containing 1.75% of chromium and weighing 8 tons is to be cast, how much ferro-chrome containing 65% of chromium must be used?

40. Divide 47,632 by $3\frac{1}{3}$. Multiply 237.93 by $12\frac{1}{2}$.

41. When Adjutant Casala, the French aviator, attained a record altitude of 10,000 meters, what was his altitude expressed in feet?

42. Determine the area of a field whose dimensions as indicated in the accompanying diagram are given in feet. Give the answer correct to the nearest tenth of an acre.



43. Determine the weight of 1000 cast-brass cylindrical cups $2\frac{1}{4}$ inches in diameter, outside, $2\frac{1}{2}$ inches high, with walls and bottom $\frac{1}{8}$ inch thick, if brass weighs 504 pounds per cubic foot. Allow 5 per cent for waste in casting.

44. If a cube 1 foot each way is cut in two diagonally, what is the total surface of each of the pieces into which the cube is cut?

45. If the standard 150-yard target has a bull's-eye 8 inches in diameter, what should be the diameter of a bull's-eye which will look the same at 25 yards?

46. Four men purchase a piece of property with the understanding that the first is to pay $\frac{1}{16}$ of the purchase price, the second to pay $\frac{3}{16}$, the third to pay $\frac{5}{16}$, and the fourth to pay $\frac{7}{16}$. If they buy the property for \$250,000 and sell it for \$800,000, what is each man's share of the profit?

47. Determine the number of bricks required for a piece of pavement 50 feet wide and 300 feet long if the bricks are 2" by 4" by 8" and laid edgewise.

48. Set forth graphically the following data concerning Alaska:

Ocean products	\$23,112,000
Mineral products	19,248,000
Miscellaneous products of soil and forest	300,000

49. If the earth's equator is considered as a belt 25,000 miles long, how much clearance would there be if the equator were lengthened two feet?

50. What is the approximate weight of a nearly spherical goldfish jar 16 inches in diameter, filled with water, allowing 8 pounds for the weight of the jar?

51. What is the total external pressure on the human body if the external surface is 16 square feet and the air pressure is 15 pounds per square inch?

52. If there are 250 men in a shop and 20 per cent of them are laid off, what is the number remaining at work?

53. If the crater of a volcano is 2 miles in diameter and its depth is $\frac{1}{4}$ mile, how many cubic yards of material will it hold?

54. If on a baseball diamond it is 90 feet between bases, how far is it from first base to third?

55. Determine the number of bricks required in building the walls of a warehouse 120 feet long, 60 feet wide, and 45 feet high, outside measurements, if the walls are 18 inches thick and no allowance is made for doors and windows.

56. If the diameter and altitude of a cone, the diameter of the base and the altitude of a cylinder, and the diameter of a sphere are each 2 feet, find the volume of each.

57. Nickel steel for automobile construction contains 3.4 per cent of nickel. If a ladle holds 3700 pounds of steel, how many pounds of nickel must be added?

58. If one side of a right triangle is 16 feet and the other side is $\frac{2}{3}$ of the hypotenuse, what is the length of the hypotenuse?

59. Make a wage table based on $87\frac{1}{2}$ cents per hour for every successive 20 minutes of an eight-hour day; that is, for 20 minutes, 40 minutes, 1 hour, etc.

60. A table top is to be constructed in the form of a hexagon. If it is to measure 4 feet 8 inches between the parallel sides of the hexagon, what will be the radius of the hexagon?

61. If a room is 22 feet long by 16 feet wide by 12 feet high, how much will it cost to plaster the walls and ceiling at 18 cents a square yard, no allowance being made for doors and windows?

62. How many gallons of water are there in a rectangular tank 12 feet long, 10 feet wide, and 7 feet deep if it is half full?

63. If a bar of silver weighs $8\frac{3}{4}$ pounds avoirdupois, what is it worth at 53 cents per Troy ounce?

64. Two baseball clubs have the following record :

CLUB	GAMES WON	GAMES LOST	GAMES PLAYED	PERCENTAGE
A	59	22	81	
B	56	24	81	

If these clubs are to play a final series of 10 games, how many of these games must A win in order to win out finally?

65. A piece of work was to have been done by 10 men in 20 days, but at the end of two days 3 men left. How long did it take the remaining 7 men to complete the work?

66. Prepare a chart or diagram by which Centigrade temperature readings between freezing and boiling are readily convertible to Fahrenheit readings.

67. Seed for sugar beets costs the farmer 11 cents per pound. Approximately $12\frac{1}{2}$ pounds are required to seed an acre. If 1000 acres are put into beets, what is the farmer's gain if he hauls 15,000 tons of beets to the factory, receiving \$ 5 per ton for 75 per cent of his crop and \$ 6 per ton for the remainder?

68. Two men form a partnership. One agrees to furnish 40 % of the capital and the other 60 %. If the first furnishes \$ 2500, what amount does the other furnish?

69. If the distance between bases on a ball field is 90 feet, what decimal part of an acre is occupied by the infield?

70. Out of a lot of 308 castings 18 were worthless; in another lot of 386 castings 22 were worthless. Which lot had the larger percentage of good castings?

71. Find the radii of the concentric circles that divide a circle 20 inches in diameter into three equal areas.

72. What is the height of a tree which casts a shadow 150 feet long at the same time a post 7 feet high casts a shadow 9 feet long?

73. If a shop has been paying its men for ten hours' work and then pays them the same day's wage for an eight-hour day, what per cent of increase does each man receive?

74. Determine the length of the diagonal of a cube if the length of each edge of the cube is 12 inches.

75. On May 30, 1914, Thomas drove a Delage at Indianapolis a distance of 500 miles in 6 hours, 3 minutes, 45.94 seconds. What was his average speed in miles per hour? In miles per minute?

76. A Bessemer converter contains 10 tons of molten iron to which it is desired to add 0.5 per cent manganese. How much ferromanganese containing 20 per cent of manganese must be added?

77. A rectangular box is 6 feet long, 4 feet wide, and 2 feet deep. Determine the dimensions of a similarly shaped box whose length is 8 feet.

78. What is the ratio of the volume of the larger box in the preceding example to the volume of the smaller?

79. A sugar factory consumes on an average 3000 tons of beets per day. If 75 per cent of the sugar in the beet is extracted, how many tons of sugar are produced per day from beets containing 15 per cent of sugar?

80. If a certain grade of tool steel contains 1.2 % carbon, how many pounds of carbon are contained in 250 pounds of this steel?

81. If sheet lead $\frac{1}{16}$ inch thick weighs 3.7 pounds per square foot, how many pounds will be required to line the four sides and bottom of a tank 3 feet 6 inches deep, 4 feet wide, and 8 feet long?

82. If bronze contains 85 per cent of copper, 9 per cent of zinc, and 6 per cent of tin, what is it worth per pound when copper is worth 40 cents, zinc 7 cents, and tin 36 cents?

83. Determine the shortest distance on the surface of a 12-inch cube between two diagonally opposite corners.



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84. If a soldering compound is made by fusing together 18 pounds of tin, 75 pounds of bismuth, 37.5 pounds of lead, and 19.5 pounds of cadmium, what is the percentage of each of the constituents?

85. If the diameter of the bull's-eye on a target is the same as the diameter of a 22-caliber bullet (.22 of an inch), how great an angle of deviation in sighting is possible, if the bullet fired is to touch the bull's-eye of a target 32 feet distant?

86. What is the value of a 30-ton carload of silver dollars if a silver dollar weighs $412\frac{1}{2}$ grains Troy?

87. Find the volume of a sphere whose diameter is 8 inches.

88. Determine the weight of a block of stone 9 feet 6 inches long, 2 feet 3 inches wide, and 2 feet 2 inches thick, if the weight of a cubic foot of the stone is 170 pounds.

89. A farmer delivers 5 tons of beets to a sugar factory and wants beet pulp for feed in exchange, the latter costing \$1.60 per ton. The tare (percentage of dirt, tops, etc.) amounts to

40 %, while the beets tested showed 15 % in sugar. The purity is found to be 80 %. Calculate the weight of pulp received by the farmer if the company pays him 2 cents for every pound of raw sugar crystallized from his beets.

90. A, B, and C ate equally of a stock of provisions which A and B furnished. The values of what A and B contributed were in the ratio of 7 to 8 respectively. C paid \$30 for what he ate. How should A and B have divided the money?

91. Two gears work together. One has 36 teeth and the other 14. How many revolutions does the smaller one make while the larger one makes 28 revolutions?

92. If cast iron weighs .26 of a pound, determine the weight of a solid spherical cast iron shot 9 inches in diameter.

93. From the answer obtained to the preceding example, determine the weight of a solid spherical cast-iron shot $4\frac{1}{2}$ inches in diameter.

94. Determine the diameter of a cast-iron sphere which will weigh 16 pounds.

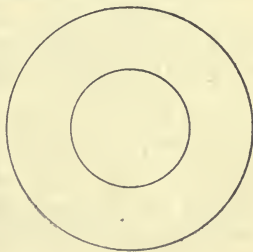
95. The top of a mountain pass is 1500 feet higher than the level of the base and is reached by a zigzag road 6 miles long. What is the average grade of the road?

96. A rectangular-shaped package is 1 meter long, 3 decimeters wide, and 8 centimeters thick. How many feet of string are required to tie it if the string goes around the package twice each way?

97. At the rate of 1000 yards a minute how long will it take a homing pigeon to fly 300 miles?

98. Find the weight of 3 pine poles 2 in. by 4 in. and 14 ft. long, if pine weighs 35 pounds per cubic foot.

99. Find the area between the two circumferences if the radius of the smaller circle is 50 feet, and that of the larger is 100 feet.



100. Find the weight of a concrete pyramid with a square base 50 feet on a side and 30 feet high, if concrete weighs 154 pounds per cubic foot.

101. What is the height of a telephone pole if the angle of elevation of the top from a point 140 feet away is 17 degrees?

102. What is the weight of a cylindrical glass jar 12 inches tall, 9 inches in diameter, and $\frac{3}{16}$ of an inch thick? (See p. 130.)

103. What is the capacity in pints of a cylindrical can $4\frac{1}{2}$ inches in diameter and $4\frac{3}{4}$ inches high?

104. What is the weight of a pane of plate glass 40" by 30" and $\frac{3}{8}$ of an inch thick, if plate glass weighs 160 pounds per cubic foot?

105. The following table gives the proportions of the most important substances in solution in sea water. All together they form about 3.5 per cent of the weight of the water.

Chloride of Sodium	.75
Chloride of Magnesium	.10
Sulphate of Magnesium	.05
Sulphate of Calcium	.04
Chlorate of Potash	.04
Bromide of Sodium	.01
Carbonate of Calcium	.005
All other substances	.005

Make a graph setting forth the above facts.

106. At 40 cents a pound for steel, find the cost of material to make 10,000 steel balls $\frac{1}{2}$ inch in diameter for ball bearings, allowing 2 per cent for waste.

107. How many ounces of gold will be required to make one dozen gold baseball watch fobs $\frac{1}{4}$ of an inch in diameter?

108. If water in a 6-inch irrigating pipe is flowing at the rate of 8 feet per minute, how many gallons of water are delivered per hour at the end of the pipe?

109. How many feet of belting are there in a roll of 40 turns where the diameter of the roll is 32 inches and the diameter of the eye is 5 inches?

110. How many gallons of oil in an upright cylindrical tank 10 feet in diameter and 10 feet high, if it is filled to a depth of 3 feet 6 inches?

111. Determine the number of gallons of water a hemispherical bowl 16 inches in diameter will contain.

112. How many feet of pipe 2 inches outside diameter, $\frac{1}{4}$ inch thick, can be made from 1 ton of lead, allowing 5 per cent for waste, if lead weighs 710 pounds per cubic foot?



ROMAN AQUEDUCT IN FRANCE

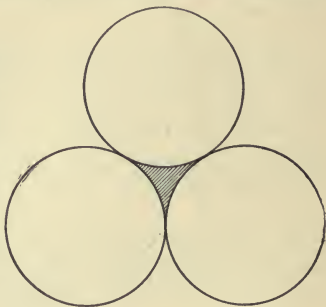
113. What is the cost of putting a 4-foot sidewalk along the front and one side of a lot 60 feet by 160 feet at 15 cents per square yard?

114. Prepare a table for the conversion of French units of length into English units of length.

115. How many cones 2 inches in diameter at the top and 5 inches deep can be filled with 1 gallon of ice cream?

116. If copper weighs 550 pounds per cubic foot, find the length of wire of average diameter, $\frac{1}{16}$ inch, that can be made from 10 pounds of copper.

117. Find the area of the shaded portion of the accompanying diagram if the radius of each of the circles is 1 mile. Express the answer correct to the nearest tenth of an acre. Express the answer in square yards, correct to the nearest square yard.



118. How many shot $\frac{1}{16}$ of an inch in diameter can be made from 1 pound of lead, allowing 5 per cent for waste?

119. A cylindrical tank 30 inches in diameter and 14 feet long lying on its side holds how many gallons when filled to a depth of 5 inches? to a depth of 10 inches? to a depth of 15 inches?

120. If a stake 8 feet high casts a shadow 4 feet 6 inches long at the same time a building casts a 200-foot shadow, what is the height of the building?

121. How many revolutions does a cart wheel 4 feet in diameter make in going half a mile?

122. Find the length of a belt to connect two pulleys, one 18 inches in diameter, the other 12 inches, if the distance between centers is 14 feet.

123. If an aeroplane ascends 4170 feet in 10 minutes, what is the rate of ascent in miles per hour?

124. If a can is 7 inches square on the bottom, how deep must it be to hold 3 gallons?

125. How many board feet of $\frac{3}{8}$ " flooring will it take for a floor 18' by 24' if allowance of $\frac{1}{4}$ is made for matching and 4 % for waste?

126. What is the weight (Troy) of one dozen plain sterling silver band bracelets if the diameter is 2.5 inches, the width .25 inch, and the thickness .06 inch, and silver weighs 655 pounds (Avoir.) per cubic foot?

127. Determine the number of feet of rope $\frac{1}{2}$ inch in diameter in a coil, if the coil measures 14 inches over all, has a 2-inch eye, and is 8 inches thick.

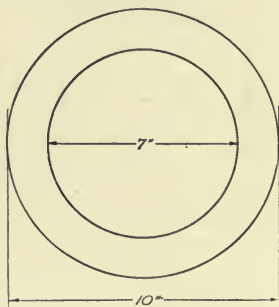
128. If a dirigible balloon is driven 500 miles in 11 hours, 43 minutes, what is the average speed in miles per hour?

129. Determine the weight of a steel plate 12" by 20" by 2", if a cubic inch of steel weighs .28 pound.

130. How many cylindrical maple pillars of diameter 8 inches and length 10 feet will it take to weigh 2 tons, if maple weighs 50 lb. per cubic foot?

131. A circular piece of lead 1 inch thick and 10 inches in diameter has a 7-inch circle cut out of it. What is the weight of the part that is left if lead weighs .41 pound per cubic inch?

132. How many glass marbles $1\frac{1}{2}$ inches in diameter can be made of 15 pounds of material, allowing 5 % for waste?



133. What is the capacity in gallons, in bushels, and in cubic feet of a cylindrical barrel 4 feet high and $2\frac{1}{2}$ feet in diameter?

134. If a tractor will draw a farm implement at the rate of a kilometer in 15 minutes, what is its speed in feet per second?

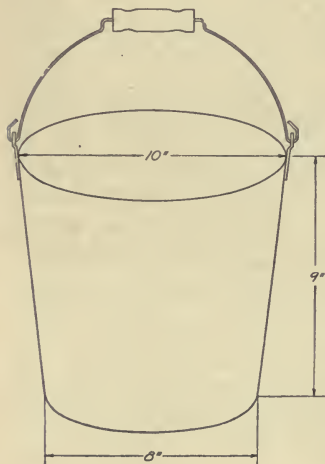
135. How long a section of a take-down rifle can be packed on the bottom of a trunk 3 feet by 2 feet by 2 feet? How long if placed diagonally?

136. What is the surface speed of an 8-inch emery wheel if its pulley is 4 inches in diameter and is driven by an 18-inch pulley making 220 revolutions per minute?



TRACTOR-DRAWN FARM IMPLEMENTS

- 137.** What length of wire 1 millimeter in diameter can be wound on a spool 5 centimeters in diameter, whose shank is 1 decimeter long and 1 centimeter in diameter?



- 138.** How long will it take a tractor-drawn mower to travel 1 mile if it travels at the rate of $3\frac{3}{4}$ miles an hour?

- 139.** Find the capacity in gallons and tenths of a gallon of a pail with the dimensions given in the figure. What is its capacity in liters? What is the capacity of a pail similar in shape which has each of its dimensions twice that of the original pail?

140. What is the percentage of error in calling a meter 39.37 inches ?

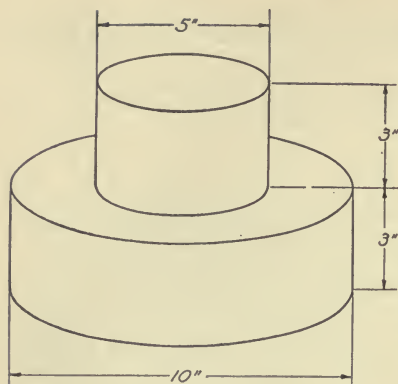
141. What is the actual difference in time between two places whose longitudes are 120° and 122° ?

142. What is the area of a regular octagon made by cutting off the corners of a square each of whose sides is 12 inches ?



ECHO BRIDGE OVER CHARLES RIVER NEAR BOSTON

143. If a person pronounces 8 syllables per second, and sound travels 1100 feet per second, at what distance from the face of a precipitous cliff must he stand in order that the echo may be heard to repeat the last four syllables ?



144. Find the weight of an iron casting of the dimensions given in the accompanying figure if cast iron weighs .26 lb. per cubic inch.

145. On a certain map the linear scale is 1 in. to 10 mi. How many acres are represented by a square $\frac{3}{4}$ in. on a side?

146. Find the area of a gravel walk 3 ft. 6 in.

wide, which surrounds a rectangular grass-plot 40 ft. long and 25 ft. wide. How wide would the walk have to be if its area were equal to the area of the grass-plot?

147. What is the perimeter of a square field that contains just one acre?

148. Obtaining the data wherever you can, prepare a graphical chart showing the advance in the prices of foodstuffs during the past four years.

149. If a lemonade glass whose bowl is hemispherical is $3\frac{1}{2}$ inches in diameter, how many gallons of lemonade will be required to fill each of 48 glasses, 6 times?

150. If a circular cake 7 inches in diameter and $3\frac{1}{2}$ inches high sells for 50 cents, what is the probable price of the same kind of cake 16 inches in diameter and 8 inches high?

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